RECITATION # 8

(9.8) Consider the following (Integer) LP program with consecutive 1's in the rows.

$$\text{Min} \quad c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4$$

s.t

$$
\begin{aligned}
x_2 + x_3 + x_4 &\geq 20 \\
x_1 + x_2 + x_3 + x_4 &\geq 30 \\
x_2 + x_3 &\geq 15 \\
x_3 + x_4 &\geq 10 \\
x_1, x_2, x_3, x_4 &\geq 0 \text{ and Integer}
\end{aligned}
$$

Transform this problem to a minimum cost flow problem.

(9.8)

let $y_1 = x_1$, $y_2 = x_1 + x_2$, $y_3 = x_1 + x_2 + x_3$, $y_4 = x_1 + x_2 + x_3 + x_4$

Replacing new variables in the given problem we have:

$$\text{Min} \quad c_1 y_1 + c_2(y_2 - y_1) + c_3(y_3 - y_2) + c_4(y_4 - y_3)$$

s.t

$$
\left.\begin{array}{lll}
y_4 - y_1 & \geq 20 & \} \; x_{41} \\
y_4 - & \geq 30 & x_{40} \\
y_3 - y_1 & \geq 15 & x_{31} \\
y_4 - y_2 & \geq 10 & x_{42} \\
y_1 - y_0 & \geq 0 & x_{10} \\
y_2 - y_1 & \geq 0 & x_{21} \\
y_3 - y_2 & \geq 0 & x_{32} \\
y_4 - y_3 & \geq 0 & x_{43}
\end{array}\right\} \text{Dual variables associate with each constraint.}
$$

Dual Problem

$$\text{Max} \quad 20x_{41} + 30x_{40} + 15x_{31} + 10x_{42}$$

s.t

$$
\begin{array}{ll}
-x_{41} - x_{31} + x_{10} - x_{21} & = c_1 - c_2 \\
-x_{42} + x_{21} - x_{32} & = c_2 - c_3 \\
x_{31} + x_{32} - x_{43} & = c_3 - c_4 \\
x_{41} + x_{40} + x_{42} + x_{43} & = c_4 \\
- \; x_{10} - x_{40} & = -c_1
\end{array}
$$

} Redundant Constraint that result from adding up all previous Constraints

② Tanker Scheduling problem.

A Steamship Company has contracted to deliver perishable goods between several different origin-destination pairs. Since the cargo is perishable, the customers have specified precise dates when the shipments must reach their destinations. (The cargoes may not arrive early or late). The steamship company wants to determine the minimum number of ships needed to meet the delivery dates of the shiploads.

a) Shipments Characteristics

Shipment	Origin	Destination	Delivery date
1	Port A	Port C	3
2	Port A	Port C	8
3	Port B	Port D	3
4	Port B	Port C	6

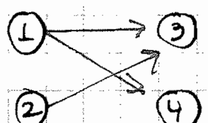
b) Shipment transit time

	C	D
A	3	2
B	2	3

c) Return times

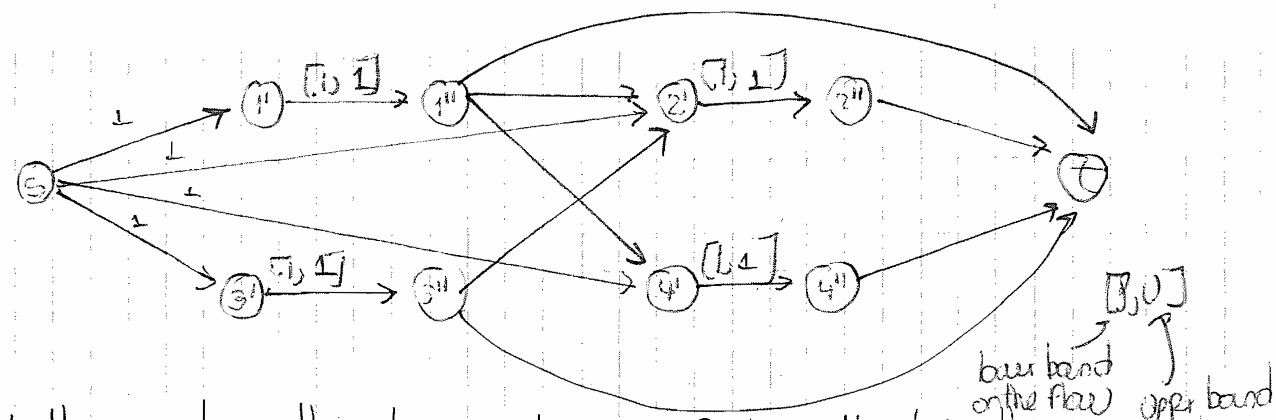
	A	B
C	2	1
D	1	2

To solve this problem construct a network in which there one node per shipment and there is an arc from node i and J if shipment J can be delivered after shipment i.



(1)

To solve the problem as a maxflow problem, split each node i in two nodes with an arc going from i' to i'' with capacity 1. Add a source node s and an arc from s to i', for all i', these arcs have unitary capacity. Add a sink t node and an arc from i'' to t $\forall i''$.
Finally if there is an arc (i,J) in (1), add an arc from i'' to J'.



Set all capacities in the network to 1. Each directed path from source s to sink t corresponds to a feasible schedule for a single ship. Then if a flow have value v, it can be decompose into schedules of v ships and our problem reduces to identify a feasible flow of minimum value.