Recitation #5

1. problem 3.7 b. from the book

   - The optimal solution is :

   $$\begin{bmatrix} 2 \\ 6 \\ 0 \\ 0 \\ 4 \end{bmatrix}$$

   - The shadow prices are the negative of the reduced cost of the slack variable associated with each constraint: $p_1 = 2, p_2 = 1, p_3 = 0$.
   - For the nonbasic variables, x_3 and x_4 we can increase the objective coefficient 2,1,0 respectively, which correspond to the negative of the corresponding reduced cost. For the basic variables we need to check when they will change the reduced cost of a nonbasic variable. For basic variable x_1 we have:

   $$\delta \cdot \tfrac{1}{2} \geq -2 \Rightarrow \delta \geq -4$$
   $$\delta \cdot \tfrac{-1}{2} \geq -1 \Rightarrow \delta \leq 2$$

   For the basic variables we need to check when they will change the reduced cost of a nonbasic variable. For basic variable x_2 we have:

   $$\delta \cdot \tfrac{1}{2} \geq -2 \Rightarrow \delta \geq -4$$
   $$\delta \cdot \tfrac{1}{2} \geq -1 \Rightarrow \delta \geq -2$$

   For the basic variables we need to check when they will change the reduced cost of a nonbasic variable. For basic variable x_5 we have:

   $$\delta \cdot \tfrac{-1}{2} \geq -2 \Rightarrow \delta \leq 4$$
   $$\delta \cdot \tfrac{1}{2} \geq -1 \Rightarrow \delta \geq -2$$

   - Range of b_1

   $$\begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{-1}{2} & \frac{1}{2} & 1 \end{bmatrix} \cdot \begin{bmatrix} 8+\delta \\ 4 \\ 6 \end{bmatrix} \geq 0$$

   Then we can increase b_1 8 units and decrease it 4 units.

   Range of b_2

   $$\begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{-1}{2} & \frac{1}{2} & 1 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 4+\delta \\ 6 \end{bmatrix} \geq 0$$

   Then we can increase b_2 4 units and decrease it 8 units.

Range of b_3

$$\begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{-1}{2} & \frac{1}{2} & 1 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 4 \\ 6+\delta \end{bmatrix} \geq 0$$

Then we can increase b_3 as much as we want and decrease it 4 units.

2. problem 3.9 from the book

Figure 1: problem 3.9

a) Finishing, $s_1 = 12.5$, sanding, $s_2 = 15$ and cutting $s_3 = 0$

b) One unit of x_3 costs $12.5 * 4 + 15 * 8 + 0 * 3 = 170$, therefore the profit of x_3 should be increased to 170. (increase by 130)

c) Let δ_2 be the change in b_2. Then, using the formula given in pg 88, we see that

$$\max_i \left\{ \frac{-\bar{b_i}}{\beta_{ik}} \middle| \beta_{ik} > 0 \right\} = -140 * \frac{1}{4} = -560$$

$$\min_i \left\{ \frac{-\bar{b_i}}{\beta_{ik}} \middle| \beta_{ik} < 0 \right\} = \{400, 0\} = 0$$

Therefore, $240 \leq b_2 \leq 800$.

d) The cost new product is $12.5 * 2 + 15 * 3 + 0 * 20 = 70$ therefore it is desirable to produce x_4.

e) Shadow price of cutting is 0, so it should not be procured. The shadow price of finishing is 12.5, so it should be procured.