

Recitation #4

2.18 Frequently linear programs are formulated with intervals constraints of the form:

$$5 \leq 6 \cdot x_1 - x_2 + 3 \cdot x_3 \leq 8$$

a. Show that this constraint is equivalent to the constraints

$$\begin{aligned} 6 \cdot x_1 - x_2 + 3 \cdot x_3 + x_4 &= 8 \\ 0 \leq x_4 &\leq 3 \end{aligned}$$

The given interval constraint can be written in two constraints as:

$$5 \leq 6 \cdot x_1 - x_2 + 3 \cdot x_3$$

$$6 \cdot x_1 - x_2 + 3 \cdot x_3 \leq 8$$

And adding slack and surplus variables respectively we have:

$$6 \cdot x_1 - x_2 + 3 \cdot x_3 - s_1 = 5$$

$$6 \cdot x_1 - x_2 + 3 \cdot x_3 + x_4 = 8$$

$$s_1 \geq 0, x_4 \geq 0$$

Let us replace the first constraint by a linear combination of the first and second constraint given by (constraint 2 - constraint 1):

$$x_4 + s_1 = 3$$

Then we obtained an equivalent system of inequalities given by:

$$x_4 + s_1 = 3$$

$$6 \cdot x_1 - x_2 + 3 \cdot x_3 + x_4 = 8$$

$$s_1 \geq 0, x_4 \geq 0$$

Given that s_1 only appear in the first equation and is nonnegative, it can be consider as a slack variable and we can obtain an equivalent system of inequalities given by:

$$x_4 \leq 3$$

$$6 \cdot x_1 - x_2 + 3 \cdot x_3 + x_4 = 8$$

$$x_4 \geq 0$$

b. Indicate how the general interval linear program

$$\begin{aligned} \max z &= \sum_{j=1}^n c_j \cdot x_j \\ \text{s.t} \\ b'_i &\leq \sum_{j=1}^n a_{ij} \cdot x_j \leq b_i \quad (i = 1, \dots, m) \\ x_j &\geq 0 \quad (j = 1, \dots, n) \end{aligned}$$

Can be formulated as a bounded-variable linear program with m equality constraints.

Following the same order of ideas than in part a. we have that for all i the interval inequality can be replaced by the equivalent set of inequalities:

$$\begin{aligned} \sum_{j=1}^n a_{ij} \cdot x_j + y_i &\leq b_i \\ y_i &\leq b_i - b'_i \end{aligned}$$

1. Consider the following LP programming problem with a single constraint:

$$\begin{aligned} \min \sum_{i=1}^n c_i \cdot x_i \\ \text{s.t} \\ \sum_{i=1}^n a_i \cdot x_i &= b \\ x_i &\geq 0 \quad i = 1, \dots, n \end{aligned}$$

a. Derive a simple test for checking the feasibility of this problem.

- If $b > 0$ then if there is at least one i' such that $a_{i'} > 0$ then the problem is feasible. The solution $x_{i'} = \frac{b}{a_{i'}}$ and $x_i = 0$ for all $i \neq i'$.
- If $b < 0$ then if there is at least one i' such that $a_{i'} < 0$ then the problem is feasible. The solution $x_{i'} = \frac{b}{a_{i'}}$ and $x_i = 0$ for all $i \neq i'$.
- If $b = 0$, the problem is always feasible independent of the value of the a_i 's. Solution $x_i = 0$ is a feasible solution for all $i = 1, \dots, n$.

b. Assuming that the optimal cost is finite, develop a simple method for obtaining an optimal solution directly.

Optimal solution is given by:

$$z^* = \min_{\{i|a_i \cdot b > 0\}} \left\{ \frac{b \cdot c_i}{a_i} \right\}$$

Let us denote as i^* an i for which $\frac{b \cdot c_i}{a_i}$ is equal to z^* , then $x_{i^*} = \frac{b}{a_{i^*}}$ and $x_i = 0$ for all i such that $i \neq i^*$.

2. Solve the following LP using excel:

$$\begin{aligned} \max z &= 9 \cdot x_2 + x_3 - 2 \cdot x_5 - x_6 \\ \text{s.t} \\ 5 \cdot x_2 + 5 \cdot x_3 + x_4 + x_5 &= 10 \\ x_1 - 15 \cdot x_2 + 2 \cdot x_3 &= 2 \\ x_2 + x_3 + x_5 + x_6 &= 6 \\ x_j &\geq 0 \quad (j = 1, \dots, 6) \end{aligned}$$

Figure 1: Problem 3

The image shows a screenshot of an Excel spreadsheet and the Solver Parameters dialog box. The spreadsheet has columns G, H, and I. The 'Objective' cell (H1) contains the value 14. The variable cells are x1 through x6, with values 32, 2, 0, 0, 0, and 4 respectively. The constraints are listed in columns LHS and RHS:

	LHS	RHS
C1	10	10
C2	2	2
C3	6	6

The Solver Parameters dialog box is open, showing the following settings:

- Set Objective: $\$H\3
- To: Max Min Value Of: 0
- By Changing Variable Cells: $\$H\$4:\$H\9
- Subject to the Constraints:
 - $\$H\$12 = \$I\12
 - $\$H\$13 = \$I\13
 - $\$H\$14 = \$I\14
- Make Unconstrained Variables Non-Negative
- Select a Solving Method: Simplex LP