Recitation #3

2.3 a. Reduce the following system to canonical form. Identify slack, surplus, and artificial variables.

$$-2 \cdot x_1 + x_2 \leq 4 \tag{1}$$

$$3 \cdot x_1 + 4 \cdot x_2 \geq 2 \tag{2}$$

$$5 \cdot x_1 + 9 \cdot x_2 = 8 \tag{3}$$

$$x_1 + x_2 \geq 0 \tag{4}$$

$$2 \cdot x_1 + x_2 \geq -3 \tag{5}$$

$$-3 \cdot x_1 - x_2 \leq -2 \tag{6}$$

$$3 \cdot x_1 + 2 \cdot x_2 \leq 10 \tag{7}$$

$$x_1 \geq 0 \quad x_2 \geq 0$$

In canonical form:

$$-2 \cdot x_1 + x_2 + y_1 = 4 \tag{8}$$

$$3 \cdot x_1 + 4 \cdot x_2 - y_2 + y_3 = 2 \tag{9}$$

$$5 \cdot x_1 + 9 \cdot x_2 + y_4 = 8 \tag{10}$$

$$x_1 + x_2 - y_5 + y_6 = 0 \tag{11}$$

$$-2 \cdot x_1 - x_2 + y_7 = 3 \tag{12}$$

$$3 \cdot x_1 + x_2 - y_8 + y_9 = 2 \tag{13}$$

$$3 \cdot x_1 + 2 \cdot x_2 + y_{10} = 10 \tag{14}$$

$$x_1 \geq 0 \quad x_2 \geq 0 \quad y_i \geq 0, i = 1, \ldots, 10$$

Variables y_1, y_7, y_10 are slack variables; variables y_2, y_5, y_8 are surplus variables and y_3, y_4, y_6, y_9 are artificial variables.

b. Formulate phase I objective functions for the following systems with $x_1 \geq 0$ and $x_2 \geq 0$.

i. expressions 2 and 3 above.

$$\min y_3 + y_4$$

ii. expressions 1 and 7 above.

There is no artificial variables in expressions 1 and 7 then there is no phase I.

iii. expressions 4 and 5 above.

$$\min y_6$$

2.4 Consider the linear program

$$\begin{aligned} &\max z = x_1 \\ &\text{s.t} \\ &-x_1 + x_2 \leq 2 \\ &x_1 + x_2 \leq 8 \\ &-x_1 + x_2 \geq -4 \\ &x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

a. State the above in canonical form.

$$\begin{aligned} &\max z = x_1 \\ &\text{s.t} \\ &-x_1 + x_2 + x_3 = 2 \\ &x_1 + x_2 + x_4 = 8 \\ &x_1 - x_2 + x_5 = 4 \\ &x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0 \end{aligned}$$

b. Solve by the simplex method.
Initial basic variables: x_3, x_4, x_5.

$$\begin{aligned} -x_1 + x_2 + x_3 &= 2 \\ x_1 + x_2 + x_4 &= 8 \\ x_1 - x_2 + x_5 &= 4 \\ -z + x_1 &= 0 \end{aligned}$$

x_1 enters the basis and x_5 leaves the basis.

$$\begin{aligned} x_3 + x_5 &= 6 \\ 2 \cdot x_2 + x_4 - x_5 &= 4 \\ x_1 - x_2 + x_5 &= 4 \\ -z + x_2 - x_5 &= -4 \end{aligned}$$

x_2 enters the basis and x_4 leaves the basis.

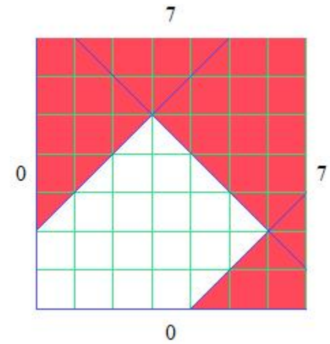
$$x_3 + x_5 = 6$$
$$x_2 + \frac{1}{2} \cdot x_4 - \frac{1}{2} \cdot x_5 = 2$$
$$x_1 + \frac{1}{2} \cdot x_4 + \frac{1}{2} \cdot x_5 = 6$$
$$-z - \frac{1}{2} \cdot x_4 - \frac{1}{2} \cdot x_5 = -6$$

Given that all coefficients of nonbasic variables in the last row (correspond to the objective function) are negative, then our last dictionary is optimal. The optimal solution is:

$$x_1^* = 6, x_2^* = 2, x_3^* = 6, x_4^* = 0, x_5^* = 0$$

c. Solve geometrically and also trace the simplex procedure steps graphically.

Figure 1: Problem 2.4c



The feasible region is shown in white.

Vertex	Lines Through Vertex	Value of Objective
(3,5)	-x+y = 2; x+y = 8	3
(0,2)	-x+y = 2; x = 0	0
(6,2)	x+y = 8; -x+y = -4	6 Maximum
(4,0)	-x+y = -4; y = 0	4
(0,0)	x = 0; y = 0	0

d. Suppose that the objective function change to $z = x_1 + c \cdot x_2$. Graphically determine the values of c for which the solution found in parts (b) and (c) remains optimal.
From the graphic in previous part, we can see that if $c \in [-1, 1]$ then optimal solution found in parts (b) and (c) remains optimal.

e. Graphically determine the shadow price corresponding to the third constraint.

From figure 2 we can conclude that the shadow price corresponding to the third constraint is 0.5.

Figure 2: Problem 2.4e