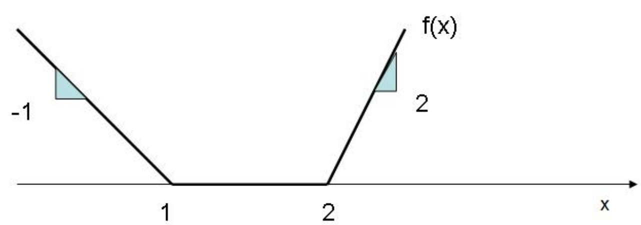
Recitation #2

1. Consider the problem of minimizing a cost function of the form $c^T \cdot x + f(d^T \cdot x)$, subject to the linear constraints $A \cdot x \geq b$. Here, d is a given vector and the function $f : \mathbb{R} \to \mathbb{R}$ is as specified in Figure 1. Provide a linear programming formulation of this problem.

Figure 1: Problem 1



Let us described $f(x)$ as:

$$\max\{-x+1, 0, 2 \cdot x - 4\} \tag{1}$$

Then a direct mathematical formulation (which is not a LP formulation) of the problem is given by:

$$\begin{array}{ll} \min & c^T \cdot x + \max\{-d^T \cdot x + 1, 0, 2 \cdot d^T \cdot x - 4\} \\ \text{subject to} & A \cdot x \geq b \end{array} \tag{2}$$

Let $v = \max\{-d^T \cdot x + 1, 0, 2 \cdot d^T \cdot x - 4\}$ then a linear programming formulation of the problem will be:

$$\begin{array}{lr} \min & c^T \cdot x + v \\ \text{subject to} & \\ & A \cdot x \geq b \\ & -d^T \cdot x + 1 \leq v \\ & 0 \leq v \\ & 2 \cdot d^T \cdot x - 4 \leq v \end{array} \tag{3}$$

2. Consider the problem:

$$\begin{array}{ll} \min & 2 \cdot x_1 + 3 \cdot |x_2 - 10| \\ \text{subject to} & |x_1 + 2| + |x_2| \leq 5 \end{array}$$

and reformulate it as a linear programming problem.

Let:

$$\begin{aligned} v &= |x_2 - 10| \\ v_1 &= |x_1 + 2| \\ v_2 &= |x_2| \end{aligned} \tag{4}$$

Then we can reformulate the problem as a linear program:

$$\begin{aligned} \min \quad & 2 \cdot x_1 + 3 \cdot v \\ \text{subject to} \quad & \\ & v_1 + v_2 \leq 5 \\ & x_2 - 10 \leq v \\ & -x_2 + 10 \leq v \\ & x_1 + 2 \leq v_1 \\ & -x_1 - 2 \leq v_1 \\ & x_2 \leq v_2 \\ & -x_2 \leq v_2 \end{aligned} \tag{5}$$

3. Write the following problem in standard form and find all basic feasible solutions and corresponding bases.

$$\begin{aligned} & \min x_1 + x_2 \\ & \text{s.t} \\ & x_1 - x_2 \leq 2 \\ & x_1 - x_2 \geq -2 \\ & x_1 \geq 0, x_2 \geq 0 \end{aligned} \tag{6}$$

In standard form we have:

$$\begin{aligned} & \min x_1 + x_2 \\ & \text{s.t} \\ & x_1 - x_2 + x_3 = 2 \\ & x_1 - x_2 - x_4 = -2 \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0 \end{aligned} \tag{7}$$

Then we have:

$$A = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 1 & -1 & 0 & -1 \end{bmatrix}$$

$$b = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

(a) Basic variables: x_1, x_4.

$$B_1 = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} \quad x_B^1 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

(b) Basic variables: x_2, x_3.

$$B_2 = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} \quad x_B^2 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

(c) Basic variables: x_3, x_4.

$$B_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad x_B^3 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$