Recitation #1

1. The Temporary Help Company must provide secretaries to its clients over the next year on the following estimates schedule: spring, 6000 secretary-days; summer, 7500 secretary-days; fall, 5500 secretary-days; and winter, 9000 secretary-days. A secretary must be trained for 5 days before becoming eligible for assignment to clients.

   There are 65 working days in each quarter, and at the beginning of the spring season there are 120 qualified secretaries on the payroll. The secretaries are paid by the company and not the client; they earn a salary of $800 a month. During each quarter, the company loses 15% of its personnel (including secretaries trained in the previous quarter).

   Formulate the problem as a linear programming problem. (Hint: Use x_t as the number of secretaries hired at the beginning of season t, and S_t as the total number of secretaries at the beginning of season t).

   Let the variables be:
   x_t Number of secretaries hired at the beginning of season t.
   S_t Number of secretaries at the beginning of season t.

   The linear program that maximizes contribution is shown in equation 1.

$$
\begin{array}{ll}
\min \quad 800 \cdot 3 \cdot (S_1 + S_2 + S_3 + S_4) & \\
\text{s.t} & \\
S_1 = 120 + x_1 & \text{No. secretaries at the beginning of the Spring} \\
65 \cdot 120 + 60 \cdot x_1 \geq 6000 & \text{Demand secretary-days for Spring} \\
S_2 = (1 - 0.15) \cdot S_1 + x_2 & \text{No. secretaries at the beginning of the Summer} \\
65 \cdot (S_2 - x_2) + 60 \cdot x_2 \geq 7500 & \text{Demand secretary-days for Summer} \\
S_3 = (1 - 0.15) \cdot S_2 + x_3 & \text{No. secretaries at the beginning of the Fall} \\
65 \cdot (S_3 - x_3) + 60 \cdot x_3 \geq 5500 & \text{Demand secretary-days for Fall} \\
S_4 = (1 - 0.15) \cdot S_3 + x_4 & \text{No. secretaries at the beginning of the Winter} \\
65 \cdot (S_4 - x_4) + 60 \cdot x_4 \geq 9000 & \text{Demand secretary-days for Winter} \\
x_t \geq 0, S_t \geq 0, t = 1, \ldots, 4. &
\end{array} \tag{1}
$$

2. The candid Camera Company manufactures three lines of cameras: the Cub, the Quickiematic and the VIP, whose contributions are $3, $9, $25, respectively. The distribution center requires that at least 250 Cubs, 375 Quickiematics, and 150 VIPs be produced each week.
   Each camera requires a certain amount of time in order to: (1) manufacture the body parts, (2) assemble the parts (lenses are purchased from outside sources and can be ignored in the production scheduling decision); and (3) inspect, test, and package the final product. The Cub takes 0.1 hours to manufacture, 0.2 hours to assemble, and 0.1 hours to inspect, test and package. The Quickiematic needs 0.2 hours to manufacture, 0.35 hours to assemble, and 0.2 hours for the final set of operations. The VIP requires 0.7, 0.1, and 0.3 hours, respectively. In addition, there are 250 hours per week of manufacturing time available, 350 hours of assembly,

and 150 hours total to inspect, test and package.
Formulate this scheduling problem as a linear program that maximizes contribution.

Let the variables be:

x_C Number of produced Cub cameras
x_Q Number of produced Quickiematic cameras
x_V Number of produced VIP cameras

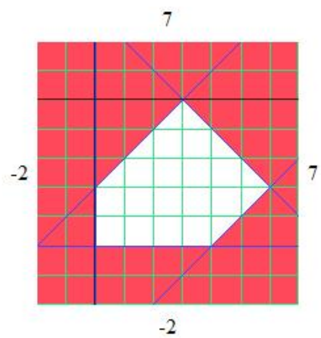
The linear program that maximizes contribution is shown in equation 2.

$$
\begin{array}{ll}
\min 3 \cdot x_C + 9 \cdot x_Q + 25 \cdot x_V & \\
\text{s.t} & \\
0.1 \cdot x_C + 0.2 \cdot x_Q + 0.7 \cdot x_v \leq 250 & \text{Manufacturing time constraint} \\
0.2 \cdot x_C + 0.35 \cdot x_Q + 0.1 \cdot x_v \leq 350 & \text{Assembly time constraint} \\
0.1 \cdot x_C + 0.2 \cdot x_Q + 0.3 \cdot x_v \leq 150 & \text{Inspect, test and package time constraint} \\
x_C \geq 250 & \text{Demand constraint for Cub cameras} \\
x_Q \geq 375 & \text{Demand constraint for Quickiematic cameras} \\
x_V \geq 150 & \text{Demand constraint for VIP cameras} \\
x_C, x_Q, x_V \geq 0 &
\end{array}
\tag{2}
$$

3. Solved geometrically.

$$
\begin{array}{l}
\max z = x_1 \\
\text{s.t} \\
-x_1 + x_2 \leq 2 \\
x_1 + x_2 \leq 8 \\
-x_1 + x_2 \geq -4 \\
x_1 \geq 0 \\
x_2 \geq 0
\end{array}
\tag{3}
$$

Figure 1: Problem 3



The feasible region is shown in white.

Vertex	Lines Through Vertex	Value of Objective
(3,5)	-x+y = 2; x+y = 8	3
(0,2)	-x+y = 2; x = 0	0
(6,2)	x+y = 8; -x+y = -4	6 Maximum
(4,0)	-x+y = -4; y = 0	4
(0,0)	x = 0; y = 0	0