4. Let $f_j(A)$ be the optimal profit the vendor can make in days $j, j+1, \ldots, N$, assuming he starts day j in location A. Similarly, let $f_j(B)$ be the optimal profit in days j through N assuming he starts day j in location B. Then,

$$f_{j-1}(A) \ = \ A_{j-1} + \max\{f_j(A), f_j(B) - c\} \tag{5}$$

and

$$f_{j-1}(B) \ = \ B_{j-1} + \max\{f_j(A) - c, f_j(B)\} \tag{6}$$

We can define 2 nodes A^i and B^i for each day i. We can have edges (A^i, A^{i+1}), (A^i, B^{i+1}), (B^i, A^{i+1}), and (B^i, B^{i+1}), with the corresponding "weights" A_i, $A_i - c$, $B_i - c$, and B_i. Our goal is to find the longest path from the "origin nodes" $\{A^1, B^1\}$ to the "destination nodes" $\{A^N, B^N\}$. The longest of these four longest paths is the optimal profit and the edges in the path give us an optimal policy. (We can negate all the weights and look for a shortest path!)

It is clear that if $D < 0$ there is no need to switch (because switching would incur a cost). One may think that if $D > c$ it always pays to switch. But that is not true. Suppose $c = 100$, and suppose $A_i = 0, B_i = 101$, and $A_{i+1} = 101, B_{i+1} = 0$. Since $B_i - A_i$ exceeds c, the rule suggests we should switch to B on day i; similarly, we should switch to A on day $i+1$. The associated net benefit is just \$2 because we pay \$200 of the \$202 we earn as switching penalties. If we stay put at A, however, we make a profit of \$101. However, it is easy to see that if $D_i > 2c$, we should switch. This is because the additional profit we make by switching is enough to offset the cost of switching to the other location and switching back.