6. (Problem 7.22 from text) **a)** The objective function is :

$$\max \quad \sum_{ijk} c_{ij} x_{ik} x_{jk}.$$

b) Constraints:

- Every person is assigned to a car pool:

$$\sum_{k} x_{ik} = 1, \forall i$$

- The number of people in a car pool is between L and U:

$$L \leq \sum_{i} x_{ik} \leq U, \forall k.$$

c) Define new variables:

$$y_{ikl} = \begin{cases} 1, & \text{if person } i \text{ is assigned to } l\text{-th vehicle in car pool } k \\ 0, & \text{otherwise} \end{cases}$$

Then the constraint is given by:

$$y_{1kl} + y_{2kl} \leq 1.$$

d) The same variables as part c). The constraint is :

$$y_{3kl} = y_{4kl}.$$

7.

(a) The optimal dictionary is the following.

$$\begin{aligned} z &= 5 - 5x_2 - s_1 \\ x_1 &= 2.5 - .5x_2 - .5s_1 \\ s_2 &= 15 - 6x_2 - 2s_1 \end{aligned}$$

The optimal basis is $B = ([2, -4], [0, 1])$.

(b) Clearly we must have $x_1 \leq 2$ in any solution to the integer program. If $x_1 = 3$ then the first constraint would be violated. ($x_1 \leq 2$ would also imply that $x_2 + s_1 \geq 1$).

Re-solving the LP with these two added inequalities yields an integer optimal solution $x_1 = 2$, $x_2 = 0$ and $z^*_{LP} = 4$. This is optimal for the IP since $z^*_{IP} \leq z^*_{LP} = 4$

(c) We start with the initial solution $x_1 = 2.5, s_2 = 15, z = 5$. We solve the original LP with the constraint $x_1 \leq 2$ and we get the same LP and integer optimal solution described in (b). We consider solving the original LP with the constraint $x_1 \geq 3$, but this is infeasible. Hence the optimal solution is such that $x_1 = 2, x_2 = 0, s_1 = 1, s_2 = 13, s_3 = 0, z = 4$.