IEOR E4004: Deterministic Models

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September 6, 2005

Procurement

The IEOR department is considering buying three kinds of computers: a Dell at \$200 each, a Sony at \$800 each, and an IBM at \$1200 each. If a total of \$80,00 is spent on 20 machines, and if at least one of eack kind is purchased, how many machines of each type does the department buy?

Setting the rent

The manager of a real estate firm wishes to set the monthly rent for 60 newly built apartment units. Past experience tells her that if she charges \$800 per month, all apartment units will be occupied; but for each \$20 increase in rent, one apartment unit is likely to remain vacant. How much rent should she charge to maximize her monthly revenue? Suppose the maintenance costs for occupied and vacant apartments are M_o and M_v respectively, with $M_v < M_o$. How does her decision change?

Making a rectangle

You have a string that is 40 yards long, and you wish to make a rectangle with it that has the largest possible area. How?

Flexible Machine Scheduling

A DRAM (dynamic RAM) manufacturer owns a flexible machine that can operate in one of two modes. In mode 1, the machine can produce 100 16-megabit DRAM chips in six hours; in mode 2, the machine can produce 100 64-megabit DRAM chips in five hours. The machine cannot be operated for more than 60 hours in any week. Also, the week's production is stored locally, and is sold at the end of the week.

A 16-megabit DRAM chip costs $10 to manufacture and is sold for $15, while a 64-megabit chip costs $20 and is sold for $24.50. The manufacturer's policy is to invest no more than $15,000 in inventory. In addition, the market for the 16-megabit chips is such that the manufacturer can sell no more than 800 of them each week. The 64-megabit chips are more popular, and hence every manufactured chip can be sold.

How should the machine be utilized if the manufacturer wishes to maximize profit?

Facility Location

Truckco has four new customers, and wishes to locate a new warehouse dedicated to them. The positions in the Euclidean plane of the four customers are $(5, 10)$, $(10, 5)$, $(0, 12)$, and $(12, 0)$. The annual number of shipments are, respectively, 200, 150, 200, and 300. Assume that shipments to different customers cannot be combined on the same trip. (In other words, shipments travel from the warehouse to the customer and back.)

Where should the new warehouse be located if Truckco wishes to minimize the total annual distance traveled by the trucks?

Transportation

Suppose Breweries 1, 2, 3, 4 supply the alcoholic needs of 4 local hotels A, B, C, and D. The transportation cost for a barrel of beer from each brewery to each hotel is listed in Table 1; the demand at each hotel and the production capacity of each brewery are also listed there. Find the minimum-cost transportation schedule.

		Hotels				
		A	B	C	D	Capacity
Breweries	1	8	14	12	17	20
	2	11	9	15	13	10
	3	12	19	10	6	10
	4	12	5	13	8	5
	Demand	5	20	10	10	

Table 1: Data for the brewery problem

Network Design

Four wells in an offshore oilfield (called 1, 2, 3, 4) need to be connected together via a pipe network to an on-shore terminal (called 0). The various "links" that can be constructed are: 1 to 2 (cost $=$ 3), 1 to 3 (cost $=$ 7), 1 to 4 (cost $=$ 3), 1 to 0 (cost $=$ 2), 2 to 3 (cost $=$ 6), 2 to 4 (cost $=$ 8), 2 to 0 (cost $=$ 5), 3 to 4 (cost $=$ 10), and 4 to 0 (cost $=$ 4).

Find the cheapest feasible pipe network.

Help the missionaries

Two missionaries and two cannibals wish to cross from the left to the right bank of a river by a row boat. The boat can carry at most two people at a time and must be brought back for the next crossing. At no time should there be more cannibals than missionaries, since the cannibals, from force of habit, would eat the outnumbered missionaries. How can the crossing take place? What if there are three missionaries and three cannibals? Can you handle four of each?

The Jeep problem

You have a car that consumes one unit of fuel for each mile travelled. Your car has a fuel capacity of 500 units, but you wish to travel 1000 miles. To achieve this, you must establish some "storage points" along the way and stock them. How do you achieve this most effectively?

Three dimensional tic-tac-toe

Consider the three-dimensional version of the tic-tac-toe problem. Can this ever end in a draw? (We know the 2d version can, and usually does.)

Knights, Knaves, and Werewolves

Suppose you are visiting a forest in which every inhabitant is either a knight or a knave. Knights always tell the truth and knaves always lie. In addition some of the inhabitants are werewolves and have the annoying habit of sometimes turning into wolves at knight and devouring people. A werewolf can be either a knight or a knave. You interview three inhabitants: A says "I am a werewolf"; B says "I am a werewolf"; and C says "At most one of us is a knight." Classify A, B, and C.

(Separate puzzle) Suppose A says "At least one of us is a knave" and B says "C is a knight." Given that there is exactly one werewolf, and that he is a knight, who is the werewolf?

A logic puzzle

There are five different houses along a street, painted in five different colors. In each house lives a person of a different nationality. These five people drink a certain type of beverage, smoke a certain brand of cigar, and keep a certain pet. No two people own the same pet or smoke the same brand of cigar or drink the same type of beverage. Some additional facts are:

- the Norwegian lives in the first house, which is Blue
- the person in the center house drinks milk
- the owner of the yellow house smokes Dunhill
- the Dane drinks tea
- the German smokes Prince

Logic puzzle (contd.)

- the Swede's pet is a dog
- the person who smokes BlueMaster drinks beer
- the bird owner smokes Pall Mall
- the green house is to the immediate left of the red house
- the person who smokes Blend lives next to the cat owner
- the horse owner lives next to the Dunhill smoker
- the Blend smoker's neighbor drinks water

Who owns the fish? Formulate the problem as a linear mathematical program. How does your answer change if all you know is that the green house is to the left of the red house, but may not be to its immediate left?

Stable Marriage

Suppose n boys and n girls are to be paired off in marriages. A set of marriages is unstable if there is a boy and a girl who are not married to each other, but prefer each other to their actual mates. Otherwise the set of marriages is stable. Does their always exist a stable set of marriages?

Locating an electrical utility

We wish to locate an electrical utility plant to supply the needs of users residing at various locations so as to minimize the transmission losses. How can we do this?