Assigment 8 Deterministic Models

P5

a) From Section 9.5 of the textbook (pag 289) we have that an integer linear program is a linear program further constrained by the Integrality restriction

To reformulate the given problem as an integer linear program let us define the binary variables:

y_i : 1 if X_1 is equal to i. $i \in \{0,1,4,6\}$
0 otherwise.

Then the Integer linear formulation is:

$$\boxed{\begin{aligned}
&\text{Max} \quad z = x_1 + 2x_2 \\
&x_1 + x_2 \le 8 \\
&-x_1 + x_2 \le 2 \\
&x_1 - x_2 \le 4 \\
&x_2 \ge 0 \\
&x_1 = 0 \cdot y_0 + 1 \cdot y_1 + 4y_4 + 6y_6 \quad (1) \\
&y_0 + y_1 + y_4 + y_6 = 1 \\
&0 \le y_i \le 1 \qquad i = 0,1,4,6 \\
&x_2 \text{ Integer} \\
&x_1 \text{ Integer} \\
&y_0, y_1, y_4, y_6 \text{ Integer}
\end{aligned}}$$

b) Let us define the variable $\underline{\omega}$, which we will enforce to be equal to $\underline{\underline{X_1^2}}$.
Then we can write the problem with the new objetive function as

$$\begin{aligned}
&\text{Max} \quad z = \omega + 2x_2 \\
&x_1 + x_2 \le 8 \\
&-x_1 + x_2 \le 2 \\
&x_1 - x_2 \le 4 \\
&x_2 \ge 0
\end{aligned}$$

New constraint $\rightarrow$ $\omega = 0^2 \cdot y_0 + 1^2 \cdot y_1 + 4^2 y_4 + 6^2 y_6$

$x_1 = 0y_0 + 1y_1 + 4y_4 + 6y_6$

$0 \le y_i \le 1 \quad i = 0,1,4,6$
x_2 Integer
x_1 Integer
ω Integer
y_0, y_1, y_4, y_6 Integer

9.6

a) Let y_1 be a 0-1 variable that is 1 if the owner should travel to Europe and 0 otherwise.

Let X be weight of the owner, which are divide in two component to model the extra cost when total weight is above 20kg

$$\max \quad 40X - 450y_1 - 5X_2$$

s.t

$$X \le 50$$

$$X \le By_1 \qquad B \gg 0 \text{ (very large positive number)}$$

$$X = X_1 + X_2;$$

$$0 \le X_1 \le 20$$

$$0 \le X_2 \le 30$$

$$y_1 \in \{0,1\}$$

b)

$$\max \quad \underbrace{0.5X}_{\text{\# articles to be printed}} - 4 \cdot 0.1 \cdot X - 4 \cdot 0.05 \cdot X - 2 \cdot 0.025 \cdot X$$

s.t

$$10 \cdot X \le 1000$$

X Integer

c)

$$\max \quad 0.3X - 7.5\left(\frac{4}{10}X_1 + \frac{2}{10}X_2 + \frac{2}{20}X_3\right)$$

s.t

$$X = X_1 + X_2 + X_3$$

$$0 \le X_1 \le 4$$

$$4 \le X_2 \le 6$$

$$10 \le X_3 \le 8$$

$$X_1 \le By_1$$

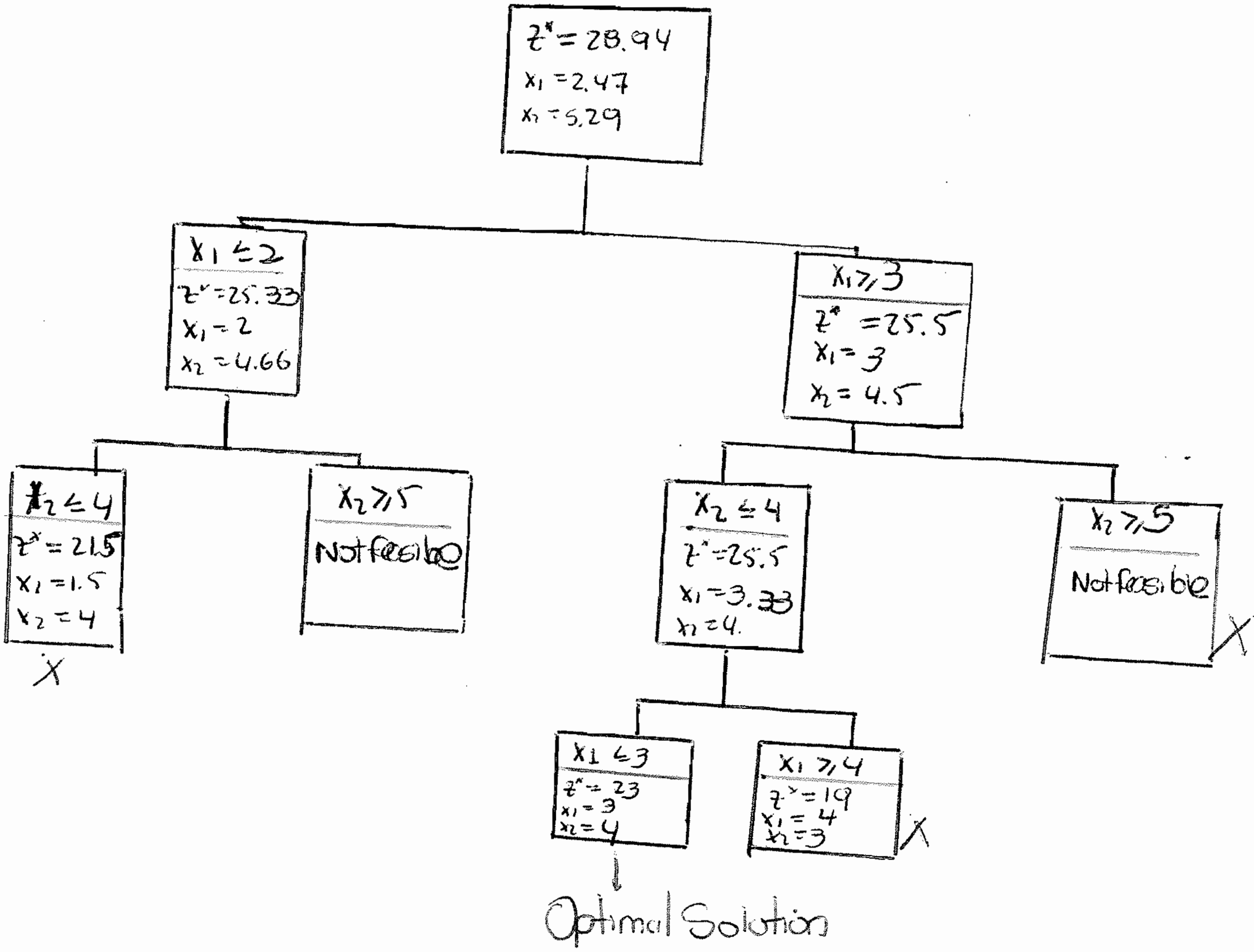
$$X_2 \le By_2$$

$$X_3 \le By_3$$

$$y_1 + y_2 + y_3 \le 1$$

$$y_1, y_2, y_3 \in \{0,1\}$$

9.10


z* = 28.94
x1 = 2.47
x2 = 5.29
x1 ≤ 2
z* = 25.33
x1 = 2
x2 = 4.66
x1 ≥ 3
z* = 25.5
x1 = 3
x2 = 4.5
x2 ≤ 4
z* = 21.5
x1 = 1.5
x2 = 4
x2 ≥ 5
Not feasible
x2 ≤ 4
z* = 25.5
x1 = 3.33
x2 = 4.
x2 ≥ 5
Not feasible
x1 ≤ 3
z* = 23
x1 = 3
x2 = 4
x1 ≥ 4
z* = 19
x1 = 4
x2 = 3
Optimal Solution

9.12

step 1 Soloing relax problem of given mixed-Integer progam we got that an optimal solution of the relax problem is:

$$x^* = (0, 0, 0, 2, 2)$$

In which x_3 and x_5 are Integer. Given that the optimal Value of the relax problem is an upper bound to the optimal value of the mixed-Integer problem and x^* is a feasible solution to the latter problem, then x^* is an optimal Soluto to the mixed-Integer problem

Note: There is a typo in the objective function, Instead of 5 it should be x_5 (the last term of the objective function)

9.10

a)

- Obtaining cut from the objective function row

$$-z + \frac{133}{5} - \frac{11}{5}x_3 - \frac{2}{5}x_4 = 0$$

$$-z + \frac{135}{5} - \frac{15}{5}x_3 - \frac{5}{5}x_4 = \frac{2}{5} - \frac{4}{5}x_3 - \frac{3}{5}x_4$$

$$\Rightarrow \quad \frac{2}{5} - \frac{4}{5}x_3 - \frac{3}{5}x_4 \leq 0$$

- Obtaining cut from first constraint

$$\frac{23}{5} - x_2 - \frac{1}{5}x_3 - \frac{2}{5}x_4 = 0$$

$$\frac{25}{5} - x_2 - \frac{5}{5}x_3 - \frac{5}{5}x_4 = \frac{2}{5} - \frac{4}{5}x_3 - \frac{3}{5}x_4$$

$$\Rightarrow \quad \frac{2}{5} - \frac{4}{5}x_3 - \frac{3}{5}x_4 \leq 0$$

- Obtaining cut from second constraint

$$\frac{16}{5} - x_1 - \frac{2}{5}x_3 + \frac{1}{5}x_4 = 0$$

$$\frac{20}{5} - x_1 - \frac{5}{5}x_3 = \frac{4}{5} - \frac{3}{5}x_3 - \frac{1}{5}x_4$$

$$\Rightarrow \quad \frac{4}{5} - \frac{3}{5}x_3 - \frac{1}{5}x_4 \leq 0$$

b)

From the feasible region of the problem we have:

$$x_3 = 11 - 2x_1 - x_2$$
$$x_4 = 6 + x_1 - 2x_2$$

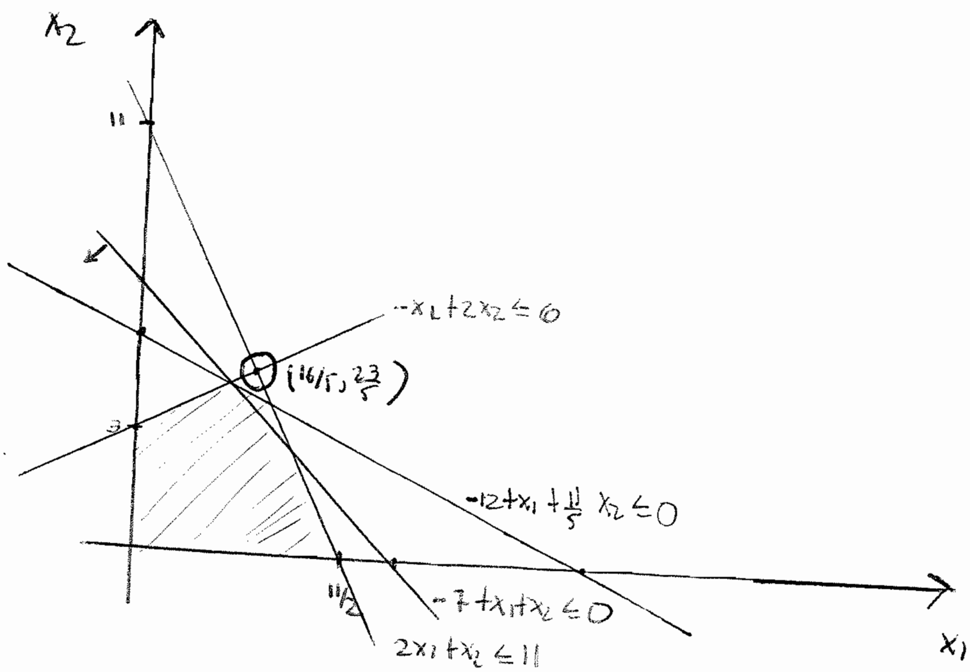
Replacing in the cuts we have:

- $\frac{2}{5} - \frac{4}{5}\left(11 - 2x_1 - x_2\right) - \frac{3}{5}\left(6 + x_1 - 2x_2\right) =$

  $-12 + x_1 + \frac{11}{5}x_2 \leq 0$

- $\frac{4}{5} - \frac{3}{5}\left(11 - 2x_1 - x_2\right) - \frac{1}{5}\left(6 + x_1 - 2x_2\right) =$

  $-7 + x_1 + x_2 \leq 0$



c)

Adding the cut obtained from the objective function:

$$x_1 + \frac{11}{5}x_2 \leq 12$$

And using excel solver we obtained the following optimal solution to the relax problem with the additional constrained described by the cut $x_1 + \frac{11}{5}x_2 \leq 12$

$$x^* = \begin{pmatrix} 3.588 \\ 3.823 \\ 0 \\ 1,94176 \end{pmatrix}$$

The solution doesnt solve the linear program, then we can proceed to find a new cut from the new optimal tableu (Relax problem with previously imposed cut)

9.15 Let $y_D, y_J, y_M, y_R, y_K, y_I$ be the 0-1 decision variables, which are associate with each one of the players and are 1 is the ~~associ~~ associate player is in the four-man basketball team and 0 otherwise.

i) $$y_K + y_I \geq 1$$

ii) $$y_J + y_M \leq 1$$

iii) $$y_D + y_J \leq 1$$

iv) $$y_I \leq B(1-y_J)$$
$$y_I \leq B(1-y_R)$$

$$\max \quad 10y_D + 9y_J + 6y_M + 6y_R + 4y_K - y_I \quad \Leftrightarrow \quad \max \frac{(10y_D + 9y_J + 6y_M + 6y_R + 4y_K \; y_D)}{4} + 5'6''$$

s.t.

$$y_K + y_I \geq 1$$
$$y_J + y_M \leq 1$$
$$y_D + y_J \leq 1$$
$$y_I \leq B(1-y_J) \quad , B \gg 0$$
$$y_I \leq B(1-y_R) \quad , B \gg 0$$
$$y_i \in \{0,1\}$$

b) Solve by Inspection