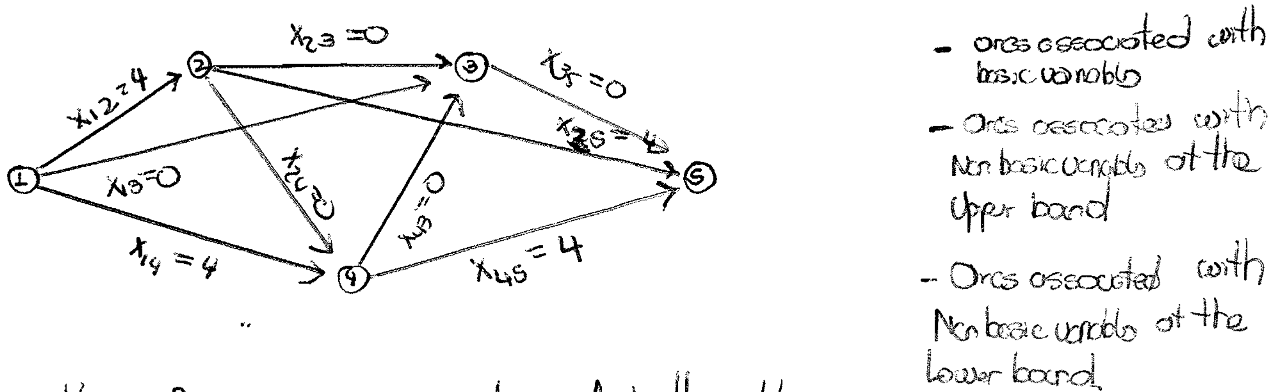
Assigment 7

8.22

a) The proposed solution is a feasible solution.
The proposed solution is a basic feasible solution.



Basic variables form a spanning tree. And all variables not in the tree are at the upper or lower bound.

b) It is a basic feasible solution

c)

$$\overline{C_{12}} = C_{12} - P_1 + P_2 = 0$$
$$\overline{C_{14}} = C_{14} - P_1 + P_4 = 0$$
$$\overline{C_{23}} = C_{23} - P_2 + P_3 = 0$$
$$\overline{C_{25}} = C_{25} - P_2 + P_5 = 0$$

Let $P_5 = 0$,

$$P_2 = C_{25} = 5$$
$$P_3 = P_2 - C_{23} = 5 - 5.5 = -0.5$$
$$P_1 = C_{12} + P_2 = 6.1 + 5 = 11.1$$
$$P_4 = P_1 - C_{14} = 11.1 - 4.4 = 6.7$$

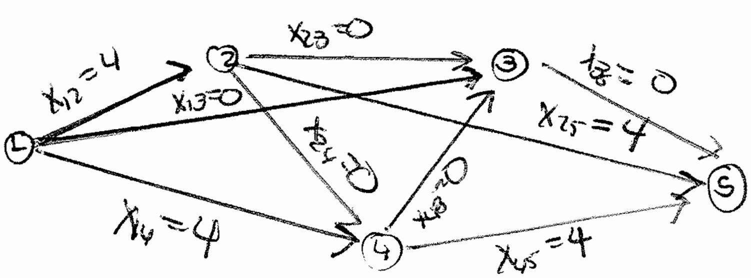
d). Basic Solution from (c) is not optimal because there is a negative cost cycle with capacity greater than zero.

Cycle $1 \to 3 \to 5 \to 2 \to 1$ has capacity 4

e) Reduced cost of Nonbasic variable at lower bound

$\overline{c_{13}} = c_{13} - p_1 + p_3 = 5 - 11.1 + (-0.5) = -6.6$

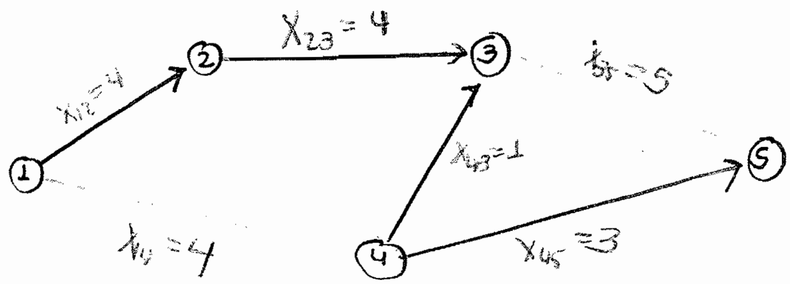
Given that x_{13} is a non basic variable at the lower bound the reduced cost indicates that is a candidate to enter the basis. Moreover given that the unique cycle created in the basic tree with the addition of x_{13} variable has capacity zero, we can arbitrarily take the arc x_{12} or x_{23} from the basic tree and add arc x_{13}. The new basic tree is:



- arcs associated with basic variables
- arcs associated with nonbasic variables at the upper bound
- arcs associated with nonbasic variables at the lower bound

8.23

a)



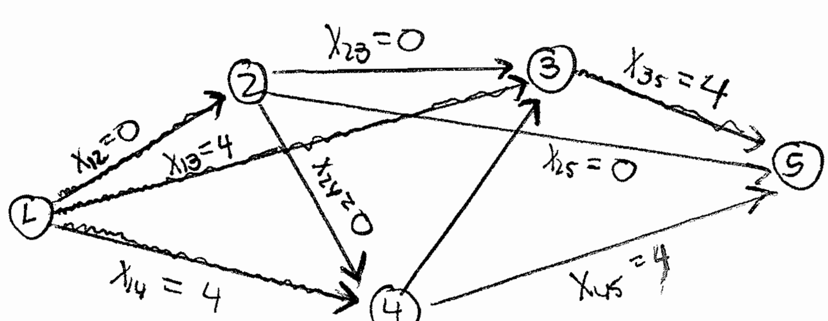
b)

$$\left.\begin{aligned} \overline{C_{12}} &= C_{12} - P_1 + P_2 = 0 \\ \overline{C_{23}} &= C_{23} - P_2 + P_3 = 0 \\ \overline{C_{43}} &= C_{43} - P_4 + P_3 = 0 \\ \overline{C_{45}} &= C_{45} - P_4 + P_5 = 0 \end{aligned}\right\} \quad \begin{aligned} &\text{Let } P_5 = 0. \\ &P_4 = C_{45} + P_5 = 3.2 + 0 = 3.2 \\ &P_3 = P_4 - C_{43} = 3.2 - 7 = -3.8 \\ &P_2 = C_{23} + P_3 = 5.5 + (-3.8) = 1.7 \\ &P_1 = C_{12} + P_2 = 6.1 + 1.7 = 7.8 \end{aligned}$$

c). This solution is not optimal. It cost is 95.6, which is higher than the cost found for the given solution in exercise 8.22, which is 74.8

d). yes, optimal solution is unique. Following solution is optimal and all non basic reduced cost are non zero. Then uniqueness of the optimal solution follows.

~~~ Arcs associate with basic Variables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9.1

let $S_i \in \{0, 1\}$ $i = 1, \ldots, 10$ and let $M \gg 0$.

i) $S_8 \leq M\,(2 - S_7 + S_1)$

ii) $S_5 \leq M\,(1 - S_3)$

$S_5 \leq M\,(1 - S_4)$

iii) $S_5 + S_6 + S_7 + S_8 \leq 2$

The Integer program to determine the minimum-cost exploration scheme that satisfies these restrictions is shown below:

$$\min \sum_{i=1}^{10} C_i S_i$$

s.t

$$S_8 \leq M\,(2 - S_7 + S_1)$$
$$S_5 \leq M\,(1 - S_3)$$
$$S_5 \leq M\,(1 - S_4)$$
$$S_5 + S_6 + S_7 + S_8 \leq 2$$
$$S_i \in \{0, 1\}$$

9.3

let $y_i \in \{0,1\}$ be a variable that Indicates if option i is selected or not. $i \in \{1,\dots,6\}$, and they are associated with the different available options.

$$\max \quad 1000{,}000\, y_1 + 200{,}000 y_2 + 300{,}000 y_3 + 400{,}000 y_4 + 450{,}000 y_5 + 450{,}000 y_6$$

s.t

$$500{,}000 y_1 + 150{,}000 y_2 + 300{,}000 y_3 + 250{,}000 y_4 + 350{,}000 y_5 + 100{,}000 y_6 \le 1{,}800{,}000$$

$$700 y_1 + 250 y_2 + 200 y_3 + 200 y_4 + 300 y_5 + 400 y_6 \le 1500$$

$$200 y_1 + 100 y_2 + 100 y_3 + 100 y_4 + 100 y_5 + 1000 y_6 \le 1200$$

$$y_6 \le y_4 + y_5 \quad \} \text{ Constraint } i$$

$$y_2 + y_3 \le 1 \quad \} \text{ Contraint } 2$$

$$y_i \in \{0,1\} \quad i = 1,\dots,6$$

9.4

a) $y_{ij}^{K} \in \{0,1\}$ is an Integer Variable that is 1 if item i is schedule before item J in machine K.

X_{iJ} : Starting time of processing item i on machine J.

$$① \begin{cases} X_{i1} \geq 0 & i = 1,2,3 \\ X_{i2} \geq X_{i1} + t_{i1} & i = 1,2,3 \\ X_{i3} \geq X_{i2} + t_{i2} & i = 1,2,3 \end{cases}$$

Item may not start processing on machine (J+1) unless it has completed on machine J.

$$② \left\{ X_{JK} - (X_{iK} + t_{iK}) \geq -M(1 - y_{iJ}^{K}) \right. \quad K = 1,2,3,\ i = 1,2,3,\ J = 1,2,3,\ i \neq J$$

if Job i is Scheduled before Job J in machine K, then job J can just start to be processin machine K after Job i is totally processed by machine K.

Min V

s.t

$V \geq X_{i3} + t_{i3}$ $\quad i = 1,2,3$ (items)

①

②

$X_{iJ} \geq 0$ $\quad i = 1 \ldots 3,\ J = 1 \ldots 3$

$y_{iJ}^{K} \in \{0,1\}$ $\quad K = 1 \ldots 3$

$y_{iJ}^{K} = 1 - y_{Ji}^{K}$ $\quad$ if Job i is chedule before J in machine K, the Job J cannot be schedule before Job i in machine K.

b) If we want items to be processed In the same Sequence on each machine then we can add the additional set of Constraints to the formulation in part (a) for all pair of Jobs i, J. $\quad i = 1 \ldots 3,\ J = 1 \ldots 3,\ i \neq J$.

$$y_{iJ}^{1} = y_{iJ}^{2} = y_{iJ}^{3}$$

9.9

a) let x_{iJKg} equal to the amount to be sent from plant i through warehouse J to customer K of product g

let z_i, y_J be $\{0,1\}$ variables that indicate if plant i is opened and warehouse J is opened respectively

$$\sum_i \sum_J X_{iJKg} = d_K^g , \forall K,g$$

Demand of customer K of product g is equal to the sum over all plants and all warehouses of the amount of product g destinate to customer K produced in plant i and send through warehouse J.

$$\sum_J \sum_K X_{iJKg} \le z_i \left(\sum_K d_{Kg} \right) , \forall i,g$$

$$\sum_i \sum_K X_{iJKg} \le y_J \left(\sum_K d_{Kg} \right) \forall J,g$$

$X_{iJKg} \geq 0$
$z_i \in \{0,1\}$
$y_J \in \{0,1\}$

In the problem there is no additional information about costs assumptions for the new model.

b) To the model in part (a) add the following constraints

$$\sum_J \sum_K X_{iJKg} \le C_i^g \quad \forall i,g$$

In which C_i^g is the maximum capacity of plant i for product g

$$L_J^g \le \sum_i \sum_K X_{iJKg} \le U_J^g \quad \forall J,g$$

In which L_J^g and U_J^g are respectively the lower and upper bound of warehouse J for product g.

c.

If no splitting order is allowed, let us define a new variable Z_{JK}, which is a 0-1 variable that is 1 if warehouse J serves customer K, 0 otherwise.
Add to the formulation in part (a) the following constraints:

$$\sum_J Z_{JK} = 1 \quad \forall K$$

$$Z_{JK} \geq X_{iJKl} \quad \forall \; i, J, K, l$$

$$Z_{JK} \in \{0, 1\}$$

d) In this part we will modify the original warehouse location model!

let $X_{ij} = X^1_{ij} + X^2_{ij}$

$$0 \leq X^1_{ij} \leq d_{ij}$$

$$X^2_{ij} \geq d_{ij}$$

And in the objective function replace every $C_{ij} X_{ij}$ term by the term $\underline{C_{ij} X^1_{ij} + C'_{ij} X^2_{ij}}$. Given that $C'_{ij} < C_{ij}$ if $X_{ij} \geq d_i$, an optimal solution will allocate all X_{ij} in X^2_{ij}.