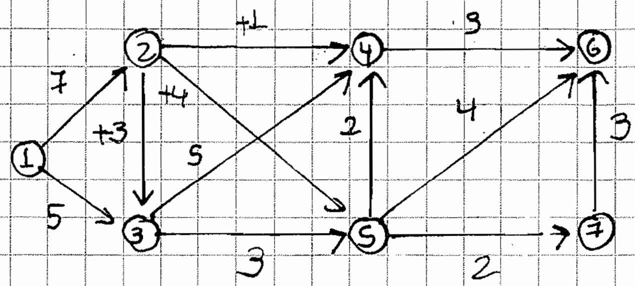
HW6 Solution



$\boxed{1}$ Interpret the numbers on arcs as distances, find the shortest path tree from node 1 to every other node in the network using Dijkstra's algorithm.

• $S = \phi$, $\bar{S} = N$, $N = \{1, \dots, 7\}$

1) let $d(1) = 0$, $d(i) = \infty$, $i \in N \setminus \{1\}$. $S = S \cup \{1\}$, $\bar{S} = \bar{S} \setminus \{1\}$

Update Node 1 adjacent list, distance labels if necessary.

$d(2) = \min\{d(1) + 7, d(2)\} = 7$

$d(3) = \min\{d(1) + 5, d(3)\} = 5$

2) Node in $\bar{S}$ with minimum distance label is 3

$S = S \cup \{3\}$, $\bar{S} = \bar{S} \setminus \{3\}$

Update node 3 adjacent list distance labels, if necessary,

$d(4) = \min\{d(3) + 5, d(4)\} = 10$

$d(5) = \min\{d(3) + 3, d(5)\} = 8$

3) Node in $\bar{S}$ with minimum distance label is 2

$S = S \cup \{2\}$, $\bar{S} = \bar{S} \setminus \{2\}$

Update node 3 adjacent list distance labels (Just to nodes that are still in $\bar{S}$) if necessary,

$d(4) = \min\{d(2) + 1, d(4)\} = 8$

4 Node in $\bar{S}$ with minimum distance label is 4 or 5, lets break ties arbitrarly, then pick 4

$S = S \cup \{4\}$, $\bar{S} = \bar{S} \setminus \{4\}$

Updates node 4 adjacent list distance labels (Just to nodes that are still in $\bar{S}$)

$d(6) = \min\{d(4) + 3, d(6)\} = 11$

5. Node in $\bar{S}$ with minimum distance label is 5

$S = S \cup \{5\}$, $\bar{S} = \bar{S} \setminus \{5\}$

(Updates node 5 adjacent list distance labels (Just to nodes that are still in $\bar{S}$)

$d(6) = \min\{d(5) + 4, d(6)\} = 11$

$d(7) = \min\{d(6) + 2, d(7)\} = 10$

6 Node in $\bar{S}$ with minimum distance label is 6

$S = S \cup \{6\}$, $\bar{S} = \bar{S} \setminus \{6\}$

Updates node 6 adjacent list distance labels (Just to nodes that are still in $\bar{S}$)

No node distance label to update

7 Node in $\bar{S}$ with minimum distance label is 7

$S = S \cup \{7\}$, $\bar{S} = \bar{S} \setminus \{7\}$. , $\bar{S} = \phi$, then stop.

Distance labels, $d(i)$, are shortest distance from 1 to i.

[2] Interpret the numbers on arcs as distances, find the shortest path tree from node 1 to every other node in the network using "Dynamic programming".

Let d_i^K be the shortest path distance from 1 to i using a path of length at most K. (path has at most K edges)

$$d_i^K = \min\{ d_i^{K-1} ,\ \min\{ d_j^{K-1} + c(j,i) : (i,j) \in A \} \}$$

Node	K=0	K=1	K=2	K=3	K=4
1	0	0	0	0	0
2	—	7	7	7	7
3	—	5	5	5	5
4	—	—	8	8	8
5	—	—	8	8	8
6	—	—	—	11	11
7	—	—	—	10	10

Nothing change from K=3 to K=4 ⇒ we can stop.

3 Interpret the number on arcs as capacities, find the maximum flow from node 1 to node 6 in that networks, using the Ford-Fulkersen algorithm. Show that what you find is indeed a maximum flow by exhibiting a minimum-cut whose capacity is the same as the value of the flow you found.

1) Send 3 units of flow through path 1 → 3 → 4 → 6

2) Send 2 units of flow through path 1 → 3 → 5 → 7 → 6

3) Send 4 units of flow through path 1 → 2 → 5 → 6

We can stop because the st cut (4, 6) (5, 6), (5, 7) is saturated, then current flow is a max flow, 9 units of flow

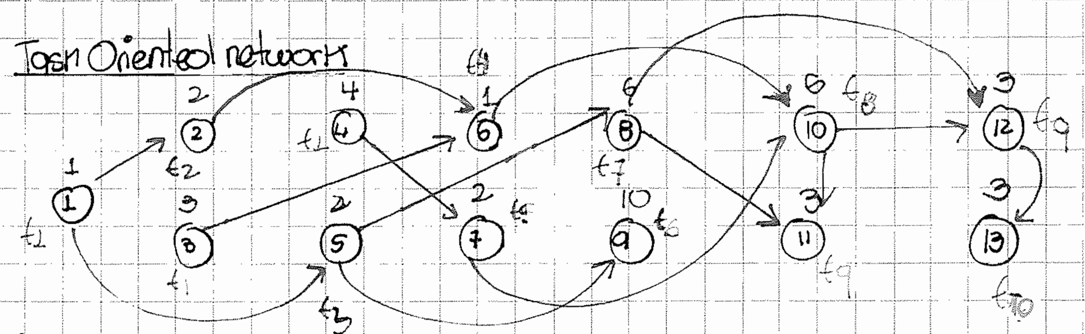
4

A Construction project involves 13 task; the tasks their estimated duration and their Immediate predecessors are show in the table below:

Task	Immediate predecessors	Duration
Task 1	—	1
Task 2	1	2
Task 3	—	3
Task 4	—	4
Task 5	1	2
Task 6	2, 3	1
Task 7	4	2
Task 8	5	6
Task 9	5	10
Task 10	6, 7	5
Task 11	8, 10	3
Task 12	8, 10	3
Task 13	12	2

a) Draw the event and task oriented networks for this problem and formulate the corresponding linear program.

Task Oriented networks



Linear program

min $(t_{10}+3) - t_1$, which is equivalent to just minimize $t_{10}-t_1$

$t_2 \geq t_1 + 1$ (P_1)

$t_3 \geq t_1 + 1$ (P_2)

$t_4 \geq t_2 + 2$ (P_3) & $t_4 \geq t_1 + 3$ (P_4)

$t_5 \geq t_1 + 4$ (P_5)

$t_6 \geq t_3 + 2$ (P_6)

$t_7 \geq t_3 + 2$ (P_7)

$t_8 \geq t_4 + 1$ (P_8) & $t_8 \geq t_5 + 2$ (P_9)

$t_9 \geq t_7 + 6$ (P_{10}) & $t_9 \geq t_8 + 5$ (P_{11})

$t_{10} \geq t_9 + 3$ (P_{12})

P_i: Dual variable associated with constraint i network problem

Dual problem

max $P_1 + P_2 + 2P_3 + 3P_4 + 4P_5 + 2P_6 + 2P_7 + P_8 + 2P_9 + 6P_{10} + 5P_{11} + 3P_{12}$

s.t

$-P_1 - P_2 - P_4 - P_5 = -1$

$P_1 - P_3 = 0$

$P_2 - P_6 - P_7 = 0$

$P_2 + P_3 + P_4 - P_8 = 0$

$P_5 - P_9 = 0$

$P_6 = 0$

$P_7 - P_{10} = 0$

$P_8 + P_9 - P_{11} = 0$ $\quad P_i \geq 0,$

[illegible] $- P_{12} = 0$ $\quad i = 1, \dots, 12$

Event Oriented Network