

IEOR E4004: Introduction to Operations Research: Deterministic Models
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HW 3 (due 02/16)

1. You are given the task of assigning D patrol officers to K highway segments. Each officer assigned to segment k reduces speeding violations on that segment by r_k . The number of officers assigned to segment k must be at least l_k and at most u_k . You are given the values of D , K , and r_k , l_k , and u_k , for $k = 1, 2, \dots, K$. Formulate a *linear* programming problem to determine an assignment of the officers to highway segments that:

- (a) maximizes total reduction achieved across all highway segments;
- (b) maximizes the *minimum* reduction achieved over all highway segments.

2. Find all alternative optima in the following dictionary:

$$\begin{array}{rcl} x_3 & = & 4 - 4x_1 - 5x_4 + 3x_5 - 2x_6 \\ x_2 & = & 2 - 2x_1 + 3x_4 - x_5 - 3x_6 \\ \hline z & = & 10 - 2x_5 - 4x_6 \end{array}$$

Note that z , x_3 and x_2 are the basic variables.

3. For the following problem, use the phase 1 procedure to find a feasible solution. Set up the dictionary to begin phase 2.

$$\begin{array}{l} \text{Max } z = 2x_1 - 3x_2 + x_3 - 4x_4 \\ \text{subject to:} \\ 2x_1 - x_2 + 3x_3 - 5x_4 \leq 20 \\ x_1 + 2x_2 - x_3 + 4x_4 \geq 2 \\ x_4 \leq 20 \\ x_1, x_2, x_3, x_4 \geq 0. \end{array}$$

4. Consider the following dictionary:

$$\begin{array}{l} x_3 = 4 + x_1 - Ax_2 \\ x_4 = 1 - Bx_1 + 4x_2 \\ x_5 = C - Dx_1 - 3x_2 \\ \hline z = 10 + Ex_1 - 2x_2 \end{array}$$

Here x_1, x_2, x_3, x_4, x_5 are the decision variables and A, B, C, D, E are unknown numbers. State conditions on A, B, C, D, E such that the following statements are true. (Each part is independent.)

- (a) The current solution is optimal. There is no other optimal solution.
- (b) The current solution is optimal. There are other optimal solutions.
- (c) The problem is unbounded.
- (d) The problem is infeasible.
- (e) The current solution is not optimal. Assume $C \geq 0$. Indicate the variable that enters the basis, the variable that leaves the basis, and what the total change in profit would be in the next iteration of the simplex method for all the values of the unknowns (for which the current solution is not optimal).

5. Problem 2.19 of the textbook.

6. Solve the linear program given in problem 2.11 of the textbook using Excel or Matlab. Attach a copy of the output.