
Nurse Scheduling Using Integer Programming

2016 Spring - IEOR E4405 Production Scheduling

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AGENDA

1 Background

2 Problem Statement

3 Mathematical Model

4 Results

5 Analysis

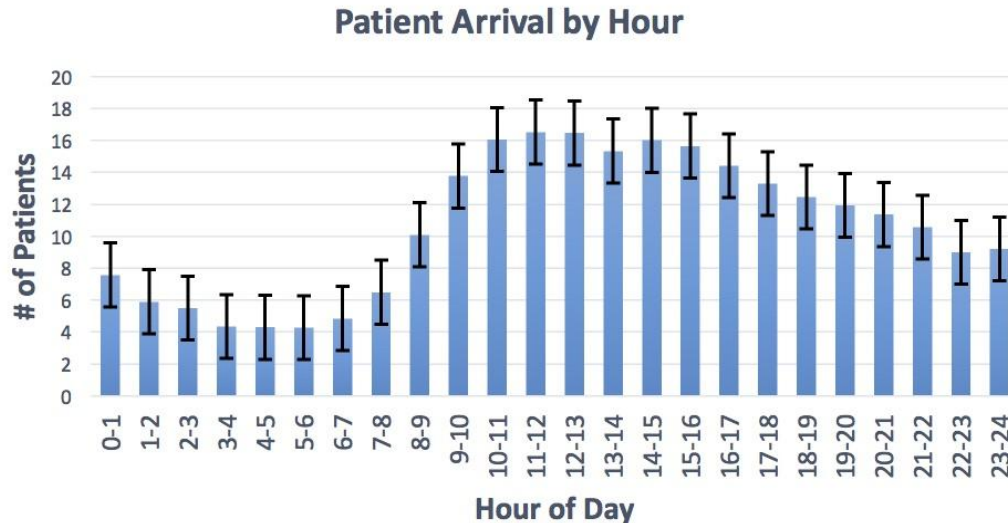
6 Limitations and Recommendations

BACKGROUND

- Scheduling nurses at hospital poses a daunting task across the industry, as many medical settings lack the necessary scheduling resources.
- Administrators are forced to resort to manual heuristics to come up with monthly schedules. Current practices, thus, lack optimality and efficiency.

PROBLEM STATEMENT

- Build an efficient scheduling method that utilizes forecasted hourly demand



IP MODEL

$$\text{Objective: } \min P_o \left(\sum_{j=1}^n \sum_{i=1}^{24} \frac{a_{ij}x_j}{d_i} - r^* \right)^+ + P_u \left(\sum_{j=1}^n \sum_{i=1}^{24} r^* - \frac{a_{ij}x_j}{d_i} \right)^+$$

$$(1) \sum_{j=1}^{10} a_{ij}x_j/d_i \leq r_o, i = 1, \dots, 24 \quad (\text{overstaffing upper bound - ratio})$$

$$(2) \sum_{j=1}^{10} a_{ij}x_j/d_i \geq r_u, i = 1, \dots, 24 \quad (\text{understaffing lower bound - ratio})$$

$$(3) \sum_{i=1}^{24} \sum_{j=1}^{10} S a_{ij}x_j \leq B \quad (\text{budget constraint})$$

$$(4) \sum_{j=1}^{10} a_{ij}x_j \geq LB, i = 1, \dots, 24 \quad (\text{minimum staffing constraint})$$

$$a_{ij} = \begin{cases} 1, & \text{if shift } j \text{ occurs during hour } i \\ 0, & \text{otherwise} \end{cases}$$

d_i nurse demand during hour i

r^* optimal nurse/demand ratio

r_o, r_u overstaffing, understaffing bounds (ratio)

B daily budget

LB minimum number of staff required during any hour

ASSUMPTIONS & DATA

Category	Figure	Source
Nurse Hourly Salary	\$33.13	Bureau of Labor Statistics (2013) ¹ - based on average income of \$68,910 per annum
Opt. Patient-to Nurse Ratio	4:1	California RN Staffing Ratio Law (2004) ² - for Emergency Departments
Minimum # Nurses on duty	9	New York Presbyterian Hospital - Interview w/ medical professional
Budget of ED	N/A	Data unavailable, assumed sufficient funds - model can account for different budget sizes
ESI* Conversion	<i>Appendix 1</i>	New York Presbyterian Hospital - Interview w/ medical professional
Patient Arrival Rates	<i>Appendix 2</i>	New York Presbyterian Hospital - Interview w/ medical professional

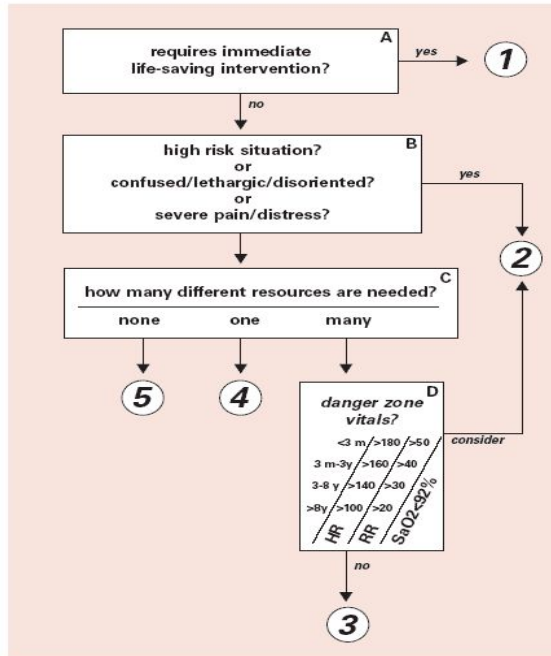
* ESI (Emergency Severity Index): Five-level category for ED nurses to triage patients according to their severity, from level 1 being the most severe to level 5

1. Bureau of Labor Statistics (2013); <http://www.bls.gov/ooh/healthcare/registered-nurses.htm#tab-5>

2. California Department of Public Health; <https://www.cdph.ca.gov/services/DPOPP/regs/Pages/N2PRegulations.aspx>

Appendix 1: ESI Conversion Algorithm

ESI Conversion Algorithm



Appendix 2: Arrival Rate Data (NYP)

Average arrival rate by ESI Level by hour of day

hour of day	ESI 1	ESI 2	ESI 3	ESI 4	ESI 5	(blank)	Grand Total
0-1	0.0096154	1.2211538	4.5673077	1.2211538	0.1826923	0.3750000	7.5769231
1-2	0.0096154	0.8750000	3.4615385	1.0192308	0.1346154	0.3942308	5.8942308
2-3	0.0288462	0.6730769	3.4807692	0.8365385	0.0769231	0.3942308	5.4903846
3-4	0.0000000	0.7307692	2.6346154	0.6923077	0.0769231	0.2115385	4.3461538
4-5	0.0096154	0.7307692	2.5576923	0.6730769	0.0961538	0.2307692	4.2980769
5-6	0.0384615	0.6923077	2.5961538	0.7019231	0.1153846	0.1250000	4.2692308
6-7	0.0192308	0.7307692	2.8653846	0.9615385	0.1153846	0.1538462	4.8461538
7-8	0.0192308	0.8942308	4.0480769	1.2403846	0.1153846	0.1730769	6.4903846
8-9	0.0576923	1.5961538	6.2788462	1.8076923	0.1730769	0.1826923	10.0961538
9-10	0.0576923	2.4326923	8.4903846	2.3942308	0.2307692	0.1634615	13.7692308
10-11	0.0673077	3.3173077	9.2884615	2.7884615	0.2980769	0.2884615	16.0480769
11-12	0.0192308	3.7307692	9.8173077	2.4423077	0.2500000	0.2692308	16.5288462
12-13	0.0673077	3.4230769	9.7980769	2.5769231	0.2692308	0.3269231	16.4615385
13-14	0.0673077	3.4134615	9.2211538	2.1634615	0.1923077	0.2788462	15.3365385
14-15	0.0865385	3.6826923	9.0769231	2.4423077	0.2403846	0.4807692	16.0096154
15-16	0.0576923	3.5192308	9.1730769	2.0865385	0.3461538	0.4711538	15.6538462
16-17	0.0384615	3.0769231	8.7788462	1.7980769	0.2884615	0.4326923	14.4134615
17-18	0.0096154	3.0000000	7.8942308	1.6346154	0.1826923	0.5673077	13.2884615
18-19	0.0192308	2.5096154	7.5769231	1.7500000	0.1826923	0.4230769	12.4615385
19-20	0.1057692	2.3653846	7.1826923	1.6442308	0.2115385	0.4326923	11.9423077
20-21	0.0384615	2.1250000	6.8461538	1.6923077	0.2403846	0.4134615	11.3557692
21-22	0.0673077	1.7884615	6.3557692	1.7307692	0.1923077	0.4326923	10.5673077
22-23	0.0000000	1.5384615	5.4711538	1.4519231	0.1442308	0.3846154	8.9903846
23-24	0.0576923	1.3173077	5.7884615	1.5096154	0.2211538	0.3173077	9.2115385
Grand Total	0.9519231	49.3846154	153.2500000	39.2596154	4.5769231	7.9230769	255.3461538

MODELING

(Sample)

Nurse Scheduling

BASE CASE

Nurse Demand

Demand Input	Hours	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
5	ESI 1	0.01	0.01	0.03	0.00	0.01	0.04	0.02	0.02	0.06	0.06	0.07	0.02	0.07	0.07	0.09	0.06	0.04	0.01	0.02	0.11	0.04	0.07	0.00	0.06
4	ESI 2	1.22	0.88	0.67	0.73	0.73	0.69	0.73	0.89	1.60	2.43	3.32	3.73	3.42	3.41	3.68	3.52	3.08	3.00	2.51	2.37	2.13	1.79	1.54	1.32
3	ESI 3	4.57	3.46	3.48	2.63	2.56	2.60	2.87	4.05	6.28	8.49	9.29	9.82	9.80	9.22	9.08	9.17	8.78	7.89	7.58	7.18	6.85	6.36	5.47	5.79
3	ESI 4	1.22	1.02	0.84	0.69	0.67	0.70	0.96	1.24	1.81	2.39	2.79	2.44	2.58	2.16	2.44	2.09	1.80	1.63	1.75	1.64	1.69	1.73	1.45	1.51
2	ESI 5	0.18	0.13	0.08	0.08	0.10	0.12	0.12	0.12	0.17	0.23	0.30	0.25	0.27	0.19	0.24	0.35	0.29	0.18	0.18	0.21	0.24	0.19	0.14	0.22
	Demand	22.66	17.26	15.94	13.06	12.86	13.09	14.73	19.77	31.28	43.13	50.43	52.30	51.69	48.53	50.20	48.84	44.81	41.00	38.48	36.89	34.79	32.13	27.21	27.89

Total	0.95
	49.38461538
	253.25
	39.25961538
	4.576923077
	788.9807692

Nurse Shifts

Decision Variables	Hours	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
x1	Shift 1								1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
x2	Shift 2																								
x3	Shift 3	1	1	1	1	1															1	1	1	1	1
x4	Shift 4								1	1	1	1	1	1	1	1	1								
x5	Shift 5																	1	1	1	1	1	1	1	1
x6	Shift 6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
x7																									
x8																									
x9																									
x10																									

Hours on Shift n	12
	5
	5
	8
	8
	8
	0
	0
	0

Budget Constraints

Budget Constraint	Hours	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Total hrs	493	Shift 1	0	0	0	0	0	0	0	20	20	20	20	20	20	20	20	20	20	20	0	0	0	0	0
Salary/hr	\$33.13	Shift 2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	14	14	14	14	14
Total salary	\$16,333	Shift 3	3	3	3	3	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Budget	\$20,000	Shift 4	0	0	0	0	0	0	0	7	7	7	7	7	7	7	7	7	7	7	0	0	0	0	0
		Shift 5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	5	5	5	5
		Shift 6	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	0	0	0	0	0
		Shift 7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		Shift 8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		Shift 9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		Shift 10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		Supply	12	12	12	12	12	9	9	36	27	27	27	27	27	27	27	25	25	25	25	19	19	19	14

Nurse Supply

Penalties

Penalty	Hours	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Pen/Nurses	1.8886	1.4383	1.3285	1.0881	1.0713	1.4541	1.6368	0.5491	1.1585	1.5976	1.8679	1.937	1.9145	1.7974	1.8593	1.9535	1.7923	1.64	1.5392	1.9418	1.831	1.6913	1.4322	1.9924	
Penalty	34.975	42.435	44.253	48.235	48.514	42.173	39.147	57.163	47.07	39.796	35.319	34.174	34.546	36.487	35.46	33.901	36.57	39.093	40.763	34.094	35.93	38.244	42.536	33.255	

Total Penalty	\$984.13
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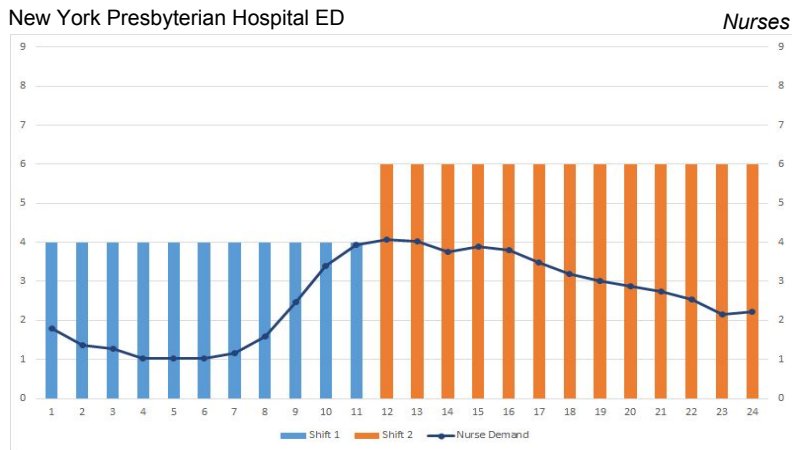
Other Constraints

Constraints	Value
Pu	\$26.50
Po	\$16.57
Max Ratio	6
Min Ratio	2
Optimal Ratio	4
Nurses	9

Objective Value

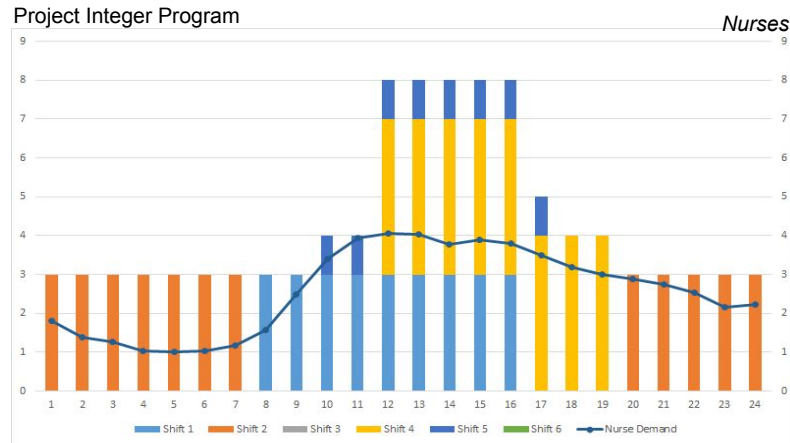
STAFFING COMPARISON

Status Quo Staffing



- Types of Shifts: 2
- Staffing: **Manual**
- Objective Value: **\$670.70**

IP Based Staffing



- Types of Shifts: 6
- Staffing: **Integer Programming**
- Objective Value: **\$485.95**

27.5%
Closer to Optimal

LIMITATIONS AND RECOMMENDATIONS

LIMITATIONS

Current implementation treats set of nurses as equal providers of care

Model optimizes schedule for staffing in a single day

RECOMMENDATIONS

Service rates: apply service rate data for various roles to stratify model

Nurse specialists: add staffing constraints for each specialty group

Simply expand set of decision variables to account for a longer time horizon

Appendix 3: Status Quo Staffing

Nurse Scheduling

BASE CASE

Demand Input	Hours	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Total
Nurses-to-Patient																										
3	ESI 1	0.01	0.01	0.03	0.00	0.01	0.04	0.02	0.02	0.06	0.06	0.07	0.02	0.07	0.07	0.09	0.06	0.04	0.01	0.02	0.11	0.04	0.07	0.00	0.06	0.95
2	ESI 2	1.22	0.88	0.67	0.73	0.73	0.69	0.73	0.89	1.60	2.43	3.32	3.73	3.42	3.41	3.68	3.52	3.08	3.00	2.51	2.37	2.13	1.79	1.54	1.32	49,384,615.88
1	ESI 3	4.57	3.46	3.48	2.63	2.56	2.60	2.87	4.05	6.28	8.49	9.29	9.82	9.80	9.22	9.08	9.17	8.78	7.89	7.58	7.18	6.85	6.36	5.47	5.79	153.25
1	ESI 4	1.22	1.02	0.84	0.69	0.67	0.70	0.96	1.24	1.81	2.39	2.79	2.44	2.58	2.16	2.44	2.09	1.80	1.63	1.75	1.64	1.69	1.73	1.45	1.51	39,259,613.88
1	ESI 5	0.18	0.13	0.08	0.08	0.10	0.12	0.12	0.17	0.23	0.30	0.25	0.27	0.19	0.24	0.35	0.29	0.18	0.18	0.21	0.24	0.19	0.14	0.22	4,576,920.77	
	Demand	8.44	6.39	5.83	4.87	4.82	4.91	5.46	7.25	11.63	16.15	19.21	20.03	19.69	18.61	19.38	18.82	17.13	15.74	14.59	14.09	13.14	12.06	10.14	10.33	298,711,538.5

Decision Variables	Hours	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Hours on Shift n
x1	4	Shift 1	1	1	1	1	1	1	1	1	1	1	1													11
x2	6	Shift 2											1	1	1	1	1	1	1	1	1	1	1	1	1	13
x3	0	Shift 3																								0
x4	0	Shift 4																								0
x5	0	Shift 5																								0
x6	0	Shift 6																								0
x7	0	Shift 7																								0
x8	0	Shift 8																								0
x9	0	Shift 9																								0
x10	0	Shift 10																								0

Budget Constraint	Hours	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Total hrs	122	Shift 1	4	4	4	4	4	4	4	4	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6
Salary/hr	\$33.13	Shift 2	0	0	0	0	0	0	0	0	0	0	0	6	6	6	6	6	6	6	6	6	6	6	6
Total salary	\$4,042	Shift 3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Budget	\$5,000	Shift 4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		Shift 5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		Shift 6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		Shift 7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		Shift 8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		Shift 9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		Shift 10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		Supply	4	4	4	4	4	4	4	4	4	4	6	6	6	6	6	6	6	6	6	6	6	6	6

Penalty	Hours	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Total Penalty
Pat/Nurses	2.1106	1.5986	1.4567	1.2163	1.2043	1.2284	1.3654	1.8125	2.9063	4.0385	4.8029	3.3381	3.2821	3.101	3.2308	3.1362	2.8558	2.6234	2.4311	2.3478	2.1907	2.0096	1.6907	1.7212		
Penalty	31.298	39.78	42.129	46.111	46.31	45.912	43.642	36.236	18.118	1.0194	21.28	10.964	11.893	14.893	12.742	14.309	18.954	22.803	25.989	27.369	29.971	32.971	38.253	37.749	5670.70	

Constraints	
Pu	\$26.50
Pb	\$16.57
Max Ratio	6
Min Ratio	2
Optimal Ratio	4
Nurses	9

Appendix 4: IP Based Staffing

Nurse Scheduling

BASE CASE

Demand input	Hours	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Total
Nurses-to-Patient																										
3	ESI 1	0.01	0.01	0.03	0.00	0.01	0.04	0.02	0.02	0.06	0.06	0.07	0.02	0.07	0.07	0.09	0.06	0.04	0.01	0.02	0.11	0.04	0.07	0.00	0.06	0.95
2	ESI 2	1.22	0.88	0.67	0.73	0.73	0.69	0.73	0.89	1.60	2.43	3.32	3.73	3.42	3.41	3.68	3.52	3.08	3.00	2.51	2.37	2.13	1.79	1.54	1.32	49.38461538
1	ESI 3	4.57	3.46	3.48	2.63	2.56	2.60	2.87	4.05	6.28	8.49	9.29	9.82	9.80	9.22	9.08	9.17	8.78	7.89	7.58	7.18	6.85	6.36	5.47	5.79	153.25
1	ESI 4	1.22	1.02	0.84	0.69	0.67	0.70	0.96	1.24	1.81	2.39	2.79	2.44	2.58	2.16	2.44	2.09	1.80	1.63	1.75	1.64	1.69	1.73	1.45	1.51	39.25961538
1	ESI 5	0.18	0.13	0.08	0.08	0.10	0.12	0.12	0.12	0.17	0.23	0.30	0.25	0.27	0.19	0.24	0.35	0.29	0.18	0.18	0.21	0.24	0.19	0.14	0.22	4.576923077
	Demand	8.44	6.39	5.83	4.87	4.82	4.91	5.46	7.25	11.63	16.15	19.21	20.03	19.69	18.61	19.38	18.82	17.13	15.74	14.59	14.09	13.14	12.06	10.14	10.33	298.7115385

Decision Variables	Hours	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Hours on Shift n
x1	3	Shift 1																								9
x2	3	Shift 2																								12
x3	0	Shift 3																								10
x4	4	Shift 4																								8
x5	1	Shift 5																								6
x6	0	Shift 6																								0
x7	0	Shift 7																								0
x8	0	Shift 8																								0
x9	0	Shift 9																								0
x10	0	Shift 10																								0

Budget Constraint	Hours	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
Total hrs	103	Shift 1																								0
Salary/hr	\$33.13	Shift 2																								3
Total salary	\$3,412	Shift 3																								0
Budget	\$4,000	Shift 4																								0
		Shift 5																								0
		Shift 6																								0
		Shift 7																								0
		Shift 8																								0
		Shift 9																								0
		Shift 10																								0
Supply		3	3	3	3	3	3	3	3	3	4	4	8	8	8	8	8	5	4	4	3	3	3	3	3	

Penalty	Hours	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Total Penalty
Pat/Nurses		2.8141	2.1314	1.9423	1.6218	1.6058	1.6378	1.8205	2.4167	3.875	4.0385	4.8029	2.5036	2.4615	2.3257	2.4231	2.3522	3.4269	3.9351	3.6466	4.6955	4.3814	4.0192	3.3814	3.4423	
Penalty		19.644	30.953	34.086	39.395	39.66	39.13	36.103	26.228	2.0706	1.0194	21.28	24.788	25.485	27.734	26.122	27.296	9.499	1.0751	5.8535	18.434	10.109	0.5097	10.247	9.2382	\$485.95

Constraints	
Pu	\$26.50
Pp	\$16.57
Max Ratio	6
Min Ratio	1
Optimal Ratio	4
Nurses	9