Airline Crew Pairing Problem

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Agenda

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- Airline Crew Pairing Problem
- Common column generation
- Advanced Column generation
- Implementation of Advanced Column generation
- Comparison
- Crew pairing considering robustness
- Row and column generation
- Conclusion

Summary

Topic

- **Airline crew pairing problem**

  = large-scale linear problem + integer programming problem

Our Project

1. **Column generation**
   1) Common column generation (Research paper 1)
   2) Advanced column generation (Research paper 2)

2. **Application of column generation**
   1) Crew pairing considering robustness
   2) Row and column generation

   (Research paper 3)

Research papers

Common column generation (Research paper 1)

- **Airline crew scheduling.**

  C. Barnhart, A.M. Cohn, E.L. Johnson, D. Klabjan, G.L. Nemhauser, and P.H. Vance. In R.W. Hall, editor, Handbook of Transportation Science, pages 517–560. Springer, 2003.

Advanced column generation (Research paper 2)

- **A new approach for crew pairing problems by column generation with and application to air transportation.**

  Lavoie, M. Minoux, and E. Odier. European Journal of Operational Research, Vol. 35, pp. 45–58, 1988.

Application of column generation (Research paper 3)

- **Solving a robust airline crew pairing problem with column generation.**

  İbrahim Mutera, Ş. İlker Birbila, Kerem Bülbüla, Güvenç Şahina, Hüsnü Yenigüna, Duygu Taşb, Dilek Tüzünc. Computers & Operations Research, Vol. 40, Issue 3, pp. 815–830, March 2013

Airline Crew Pairing Problem

Crew pairing problem

$$\min \sum_{p \in P} c_p y_p$$

$$\sum_{p:i \in p} y_p = 1 \qquad i \in F$$

$$y_p \in \{0, 1\} \qquad p \in P$$

c_p : Cost of Pair
p : Crew pairing
P : Set of all feasible pairings
i : Flight
F : Set of flight segments to be covered
y_p : 1 if pairing p is included in the solution
0 otherwise

n

m [ ]

Ex) When m= 10, n= 100
2^(m+n) = 1.07*E+301

large-scale integer liner problem
=
NP-hard

Column generation

Common column generation

Key idea of Column Generation

Since most of the variables will be non-basic and assume a value of zero in the optimal solution, only a subset of variables need to be considered.

Relaxation →Reduction of calculation →Solvable

Algorithm of Column Generation

Spep1: Make a new combination of pairs(by adding a pair)

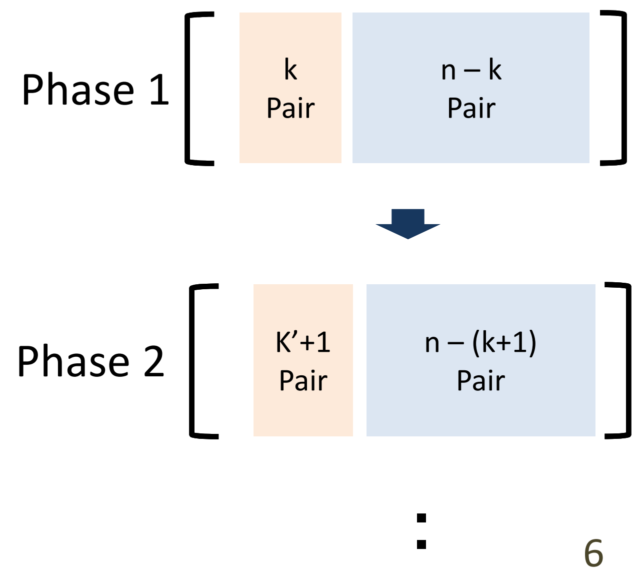
Step2: With the combination, create a linear problem (Relaxation)

Step3: Solve the problem and find optimal solution for the problem

Step 4: Check reduced cost with the optimal and decide which pairs to keep (Reduced cost = negative)

- 4-1: If no remaining pair →Step5
- 4-2: otherwise → Step1 with adding a pair

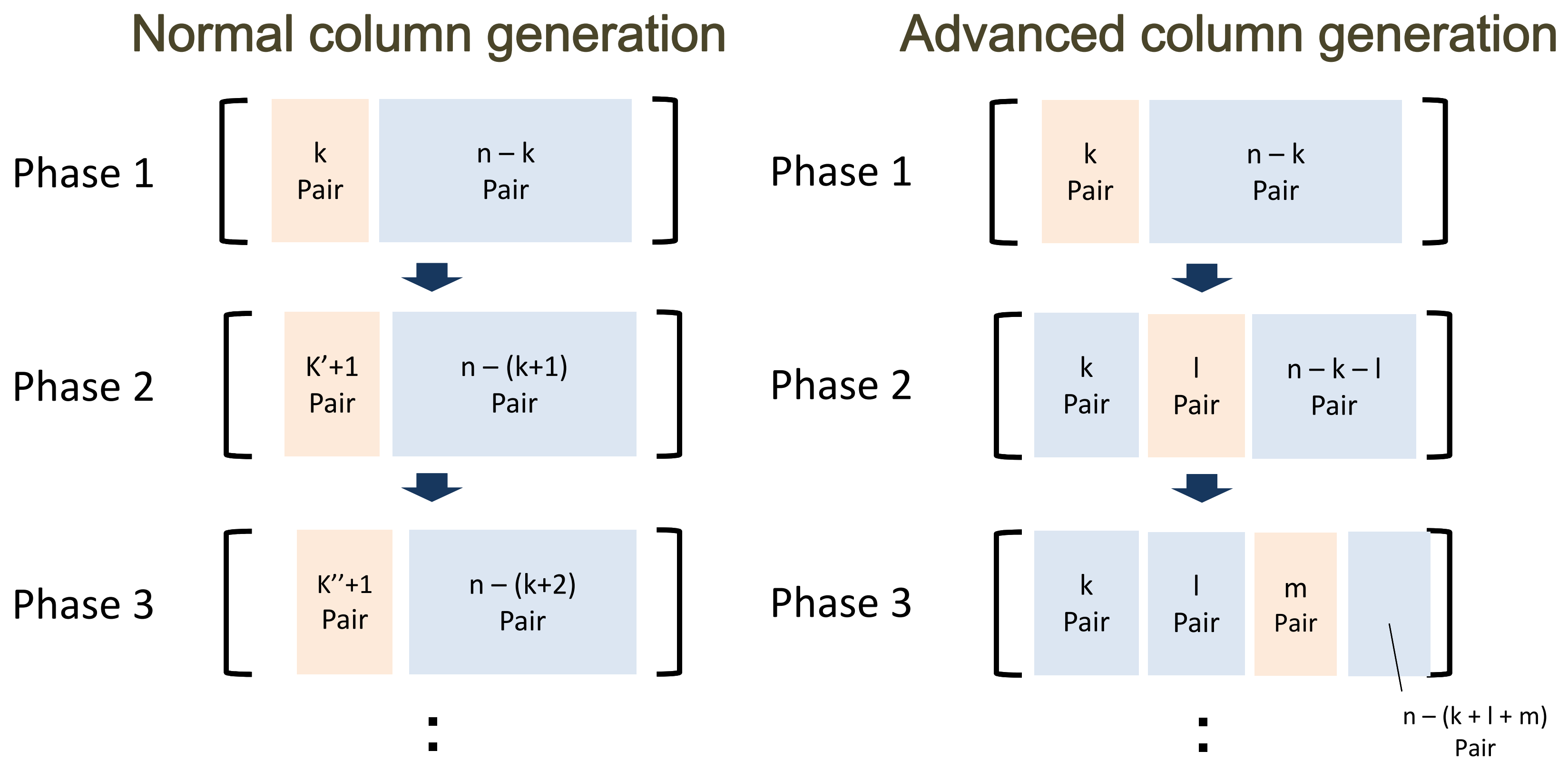
Step 5:With the pairs adopted in Step 4, solve the integer problem.



Advanced Column generation

Concept

- Basic idea is same as normal column generation
- Difference from normal column generation is in creating new combination of pairs



Implementation of Advanced Column generation

Data Condition

$$\min \ C'x$$

$$\text{s.t.} \ Ax = 1$$

C'	: Cost vector	(1000*1 matrix)
A	: Constraint matrix	(10*1000 matrix)
x	: Object variable	(1000*1 matrix)

PC Condition

- Intel(R) Core(TM) i5-2467M CPU 1.60 GHz
  1.60 GHz
- Memory: 4.00 GB
- OS: Windows 7 Home Premium

Comparison

Normal column generation	Advanced column generation
LP Relaxation ie Lower Bound 1.2000	LP Relaxation ie Lower Bound **1.2000**
IP, Initial Opt Value 17	LP Relaxation, Initial Opt Value 6.5000
LP Relaxation, Opt Value with Selected Variables 6.5000	LP Relaxation, Opt Value with Selected Variables 1.3333
IP, Opt Value with Selected Variables 14	IP, Opt Value with Selected Variables **2**
Num of Selected Variables 132	Num of Selected Variables 490
Calc Time 646.7000	Calc Time(sec) **74.4860**

Application of column generation

Crew pairing considering robustness

Case

- Extra flight is required by unexpected problem (Bad weather, problem with a Aircraft)

Definition of Robustness

- Type A: Two pairings are selected and swapped to cover an extra flight.
- Type B: One pairing with sufficient connection time between two consecutive flights is modified to cover an extra flight.

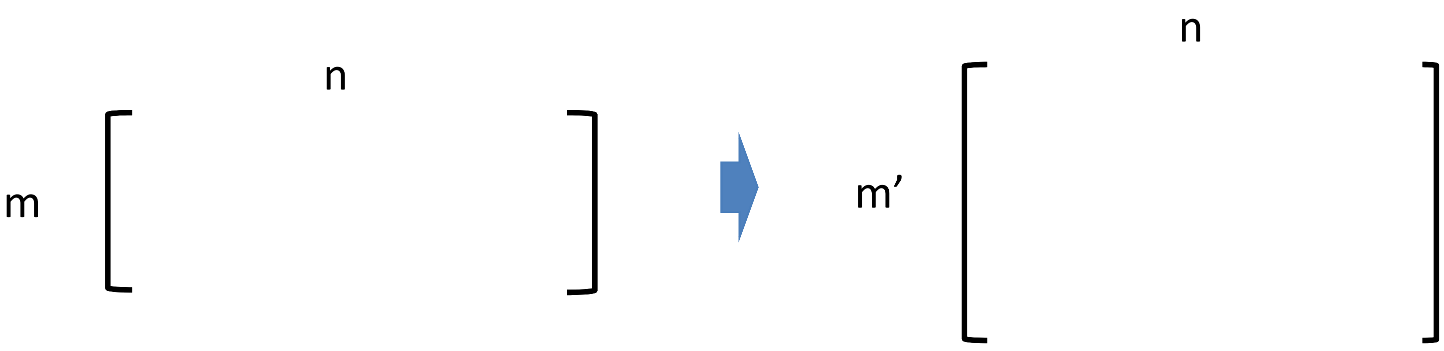
Formula (constraint)

$$\sum_{(p,q)\in\mathcal{P}_A(k)} x^k_{(p,q)} \geq \alpha_k \qquad k \in \mathcal{K}$$

K : Set of all extra flights
k : Extra flight
$P_A(k)$: Set of pairing pairs that provide a type A solution for extra flight k
α_k : Minimum number of type A solution that are required in the pairing solution
p,q: Pairing

Row and column generation

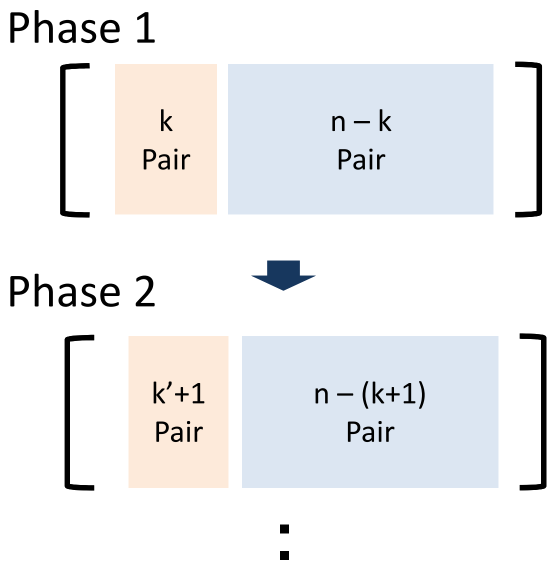
Additional constraints



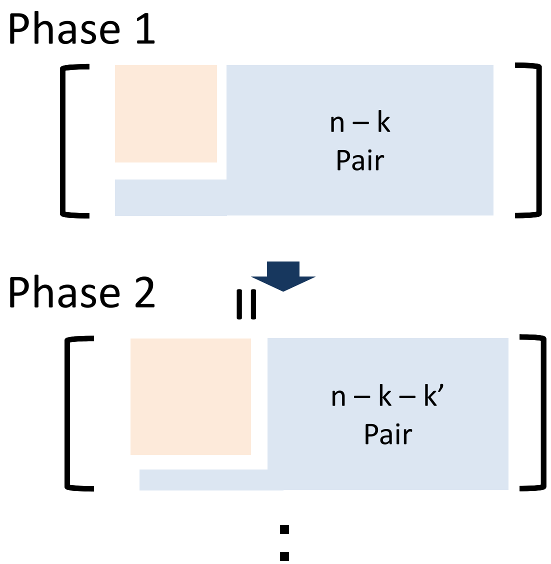
More difficult

Row and column generation

Normal column generation



Column and row generation



Conclusion

1. Column generation

1) Common column generation (Research paper 1)
2) Advanced column generation (Research paper 2)

➡ Advanced column generation gives better solution

2. Application of column generation

1) Crew pairing considering robustness
2) Row and column generation

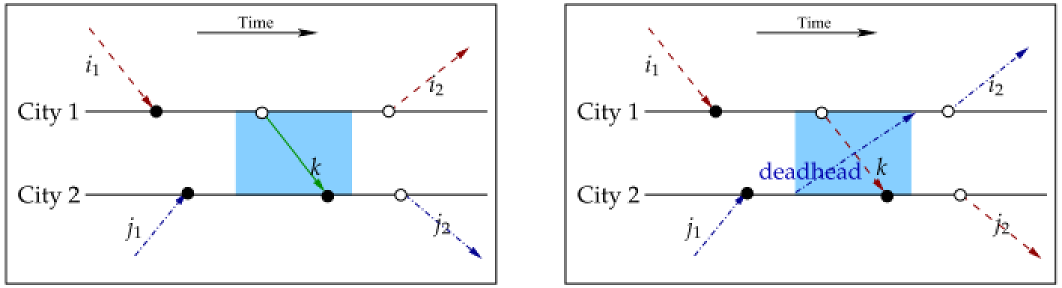
} (Research paper 3)

➡ New approach works crew pairing with robustness

Thank You

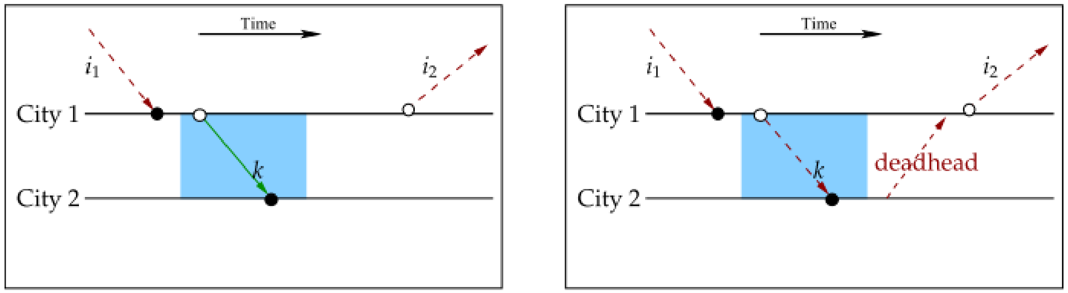
Appendix

Robustness Type A

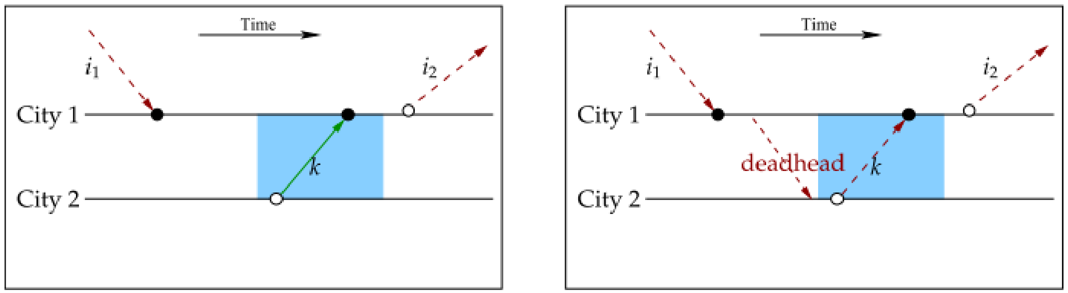


(a) Original pairings p and q. (b) Pairings p and q are partially swapped.

Robustness Type B



(a) Original pairing p. (b) Pairing p is modified to cover extra flight k.



(c) Original pairing p. (d) Pairing p is modified to cover extra flight k.

Figure 2: Extra flight k is inserted into pairing p (type B solution).