Numerial analysis of $1||\sum T_j$ using exact algorithm and heuristics

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Introduction

- Problem: $1//\Sigma T_j$
- NP-hardness
- Paper review: algorithms solving this problem
  - Simple Heuristics:
    - SPT, EDD, and **Modified Due Date(MDD)**
  - Other Heuristics:
    - Wilkerson, L.J. and Irwin (W-I)
    - Fry et al., Adjacent Pairwise Interchange (API)
    - Holsenback, J.E. and Russell, Net Benefit of Relocation (NBR)
    - **Panwalkar, Smith, and Koulamas, (P-S-K)**
    - …
  - Exact Algorithm:
    - **The Pseudo-polynomial Time Algorithm**

Pseudo-polynomial Time Algorithm

- l jobs, k = the job with the longest processing time.
- The subsets J(j, l, k): all jobs in the set {j, ..., l} with a processing time $\leq p_k$.
- V(J(j, l, k), t): the total tardiness of J(j, l, k) in an optimal sequence that starts at time t.
- **Algorithm*:**
  - Initial conditions: $V(\emptyset, t) = 0$; $V(\{j\}, t) = \max(0, t + p_j - d_j)$
  - Recursive relation:

$$V(J(j,l,k),t) = \min_{\delta}(V(J(j,k'+\delta,k'),t) + \max(0, C_{k'}(\delta) - d_{k'}) + V(J(k'+\delta+1,l,k'),C_{k'}(\delta)))$$

  - Where k‘ is such that, $p_{k'} = \max(p_{j'} \mid j' \in J(j,l,k))$
  - Optimal value function is V({1, ..., n},0).

*cited from Professor Clifford Stein's Lecture Notes

MDD Rule

- Index: $l_i(t) = (t + p_i - d_i)^+ + d_i$
- Select the next job with the smallest index value for processing.

PSK Algorithm

- Ordered set U(1,2,3,…,n) in the SPT order.
- S = the set of scheduled jobs
- p_j, d_j, $c = \sum_{i \in S} p_i$.

Step 1. If U contains only 1 job, schedule it in the last position in S and go to Step 9. Otherwise label the first job of U as the active job i.

Step 2. If $c+p_i \geq d_i$, go to Step 8.

Step 3. Select the next job of U and label it as job j.

Step 4. If $d_i \leq c + p_j$, go to Step 8.

Step 5. If $d_i \leq d_j$, go to Step 7.

Step 6. Job j becomes the active job i. If it is the last job in U, go to Step 8.Otherwise, go to Step 2.

Step 7. If j is the last job in U, go to Step 8. Otherwise, go to Step 3.

Step 8. Remove job i from U and put it in the last position in S. Set $c = c + p_i$ and go to Step 1.

Step 9. Calculate total tardiness for the sequence and stop the algorithm.

P-S-K heuristic

- Other Heuristics
  - Wilkerson-Irvin works better when EDD is optimal
  - API uses 9 switching strategies and generates 9 sequences
  - Holsenback-Russell starts with MDD, and uses net benefit of relocation

- Panwalkar et al. concluded that the P-S-K algorithm performs better than the W-I, H-R, and API heuristics for a wide range of problems, especially when due dates become tight.

Algorithm Properties

- **When works the best**
  - *MDD*
    - At most 1 job has positive tardiness or all processing times are equal
    - MDD reduces to the EDD rule when processing times are equal, and to the SPT rule when due dates are equal.
  - *PSK*
    - All jobs have positive tardiness or all due dates are equal, similar to SPT
  - *Pseudo-polynomial time algorithm*
    - Optimal
- **Cost-worst case scenario**
  - *MDD*
    - O(n log n)
  - *PSK*
    - Performs better than the pseudo-polynomial, but worse than MDD
  - *Pseudo-polynomial time algorithm*
    - $O[n^4 * \Sigma p_j]$

Computation Setup

- Implemented in *Java*
- Algorithms
  - MDD
  - PSK
  - Pseudo-polynomial time algorithm (exact algorithm)
    - Using a *recursive function* to implement the dynamic programming routine.
- Sorting is implemented by a standard *java.collections.sort* method (essentially a *mergesort* with complexity fewer than *O(nlgn)*)

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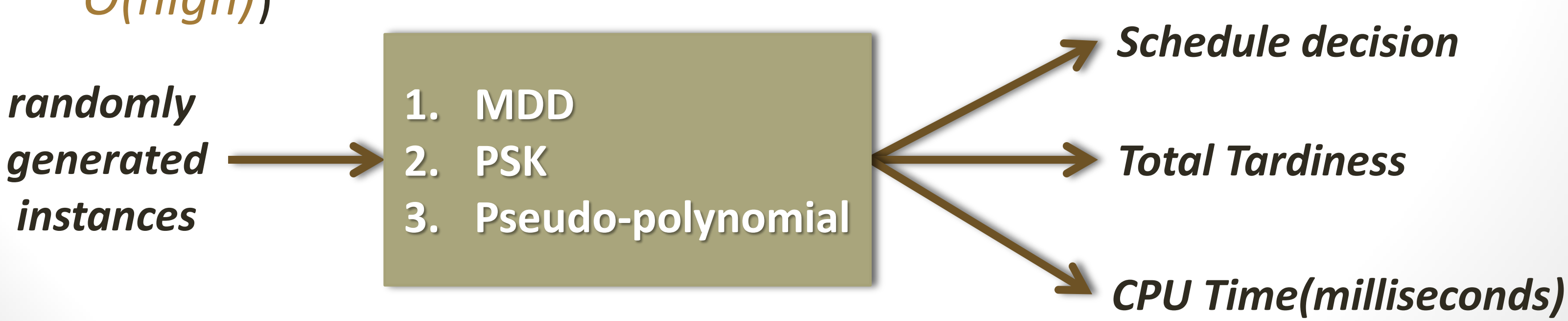
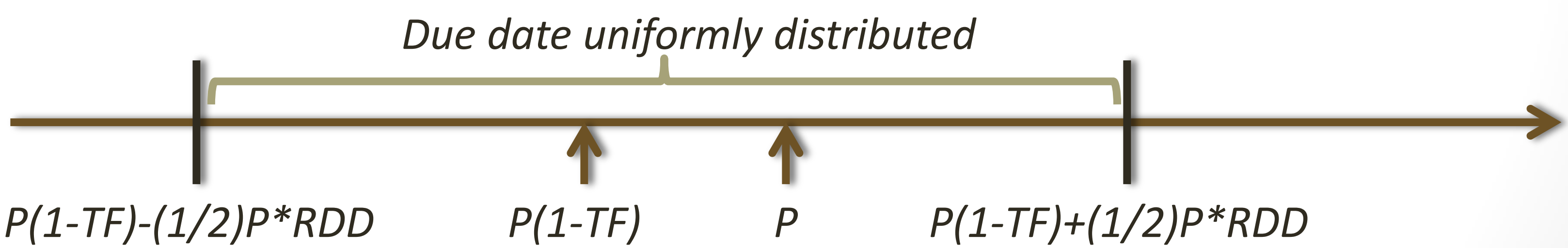


Random Instances Generation

- Generated by the method suggested by *Potts and Van Wassenhove*.
- Two instance characteristics control factors
  - RDD (Range of Due Date) {0.2,0.4,0.6,0.8,1.0}
    - Controls the variance of different due date
  - TF (Tardiness Factor) {0.2,0.4,0.6,0.8,1.0}
    - Controls the tightness of schedules



- **p_i is generated from the uniform distribution (1, p_{max}). Having computed $P = \sum_{i=1}^{n} p_i$ and selected values of RDD and TF from the set {0.2, 0.4, 0.6, 0.8, 1}, and integer due date d_i from the uniform distribution [P(1-TF-1/2*RDD), P(1-TF+1/2*RDD)] is generated for each job i.**

Results

- Performance： total tardiness and computational time
- Total tardiness of pseudo-polynomial algorithm is the benchmark of accuracy.
- Tardiness error = $\frac{tardiness - optimal\ tardiness}{optimal\ tardiness}$

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	MDD		PSK		Pseudo-polynomial Algorithm	
	tardiness error	**time**	**tardiness error**	**time**	**tardiness error**	**time**
10 jobs	0.3783	0.008	0.2728	0.112	0	0.412
15 jobs	0.4628	0.008	0.3330	0.116	0	2.260
20 jobs	0.4837	0.024	0.3362	0.072	0	45.760
25 jobs	0.5024	0.052	0.3505	0.112	0	1541.456
Mean	0.45687	0.023	0.32317	0.103	0	397.472

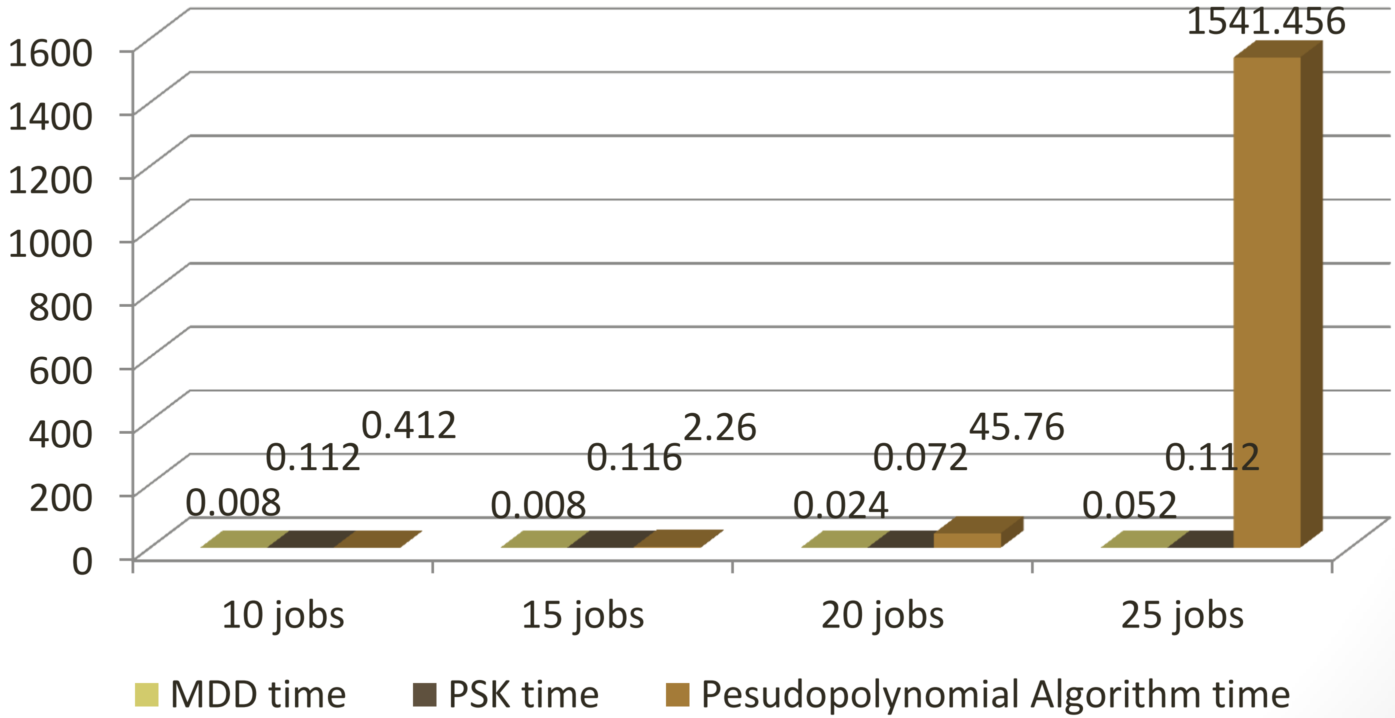
- Tardiness MDD>PSK>Pseudo-polynomial time Algorithm
- Time MDD<PSK<Pseudo-polynomial time Algorithm

Results (cont'd)

- **Tardiness error** with respect to **number of Jobs**
  - **Insight: As # jobs increases, error increases, difference between two algorithms also increases**
    - Reason: Problem becomes combinatorial complex when # of job increases. MDD starts to lose effectiveness.

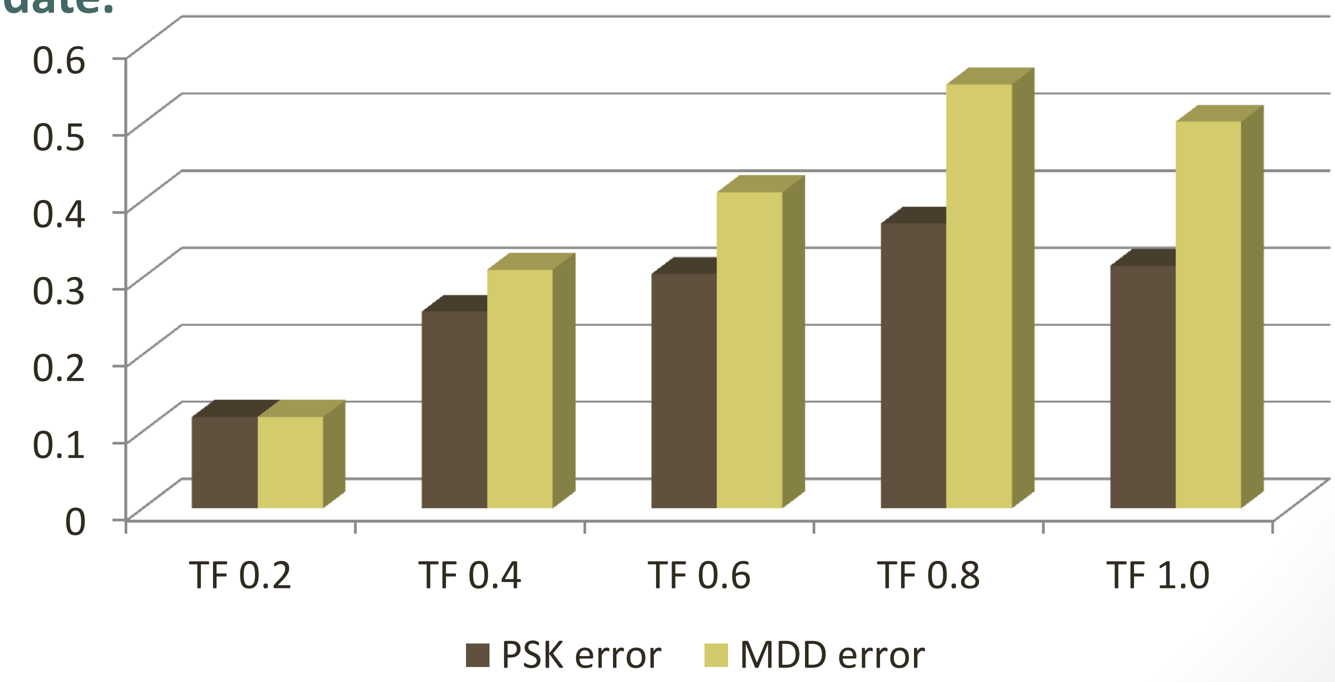
Results (cont'd)

- **Computational time** with respect to **number of jobs**
  - **Insight: CPU time of Pseudo-polynomial time algorithm is highly sensitive to number of jobs, which demonstrates the NP-hardness of the problem to solve to optimality.**



Results (cont'd)

- **Tardiness error** with respect to **Tightness of instances (TF)**
  - *10 jobs case*
  - **Insight: As TF increases, error increases, difference between two algorithms also increases. PSK starts to become much more effective compared to MDD as instances become tighter in due date.**



Conclusions

- Generally, we get
  - Tardiness MDD > PSK > Pseudopolynomial Algorithm
  - Time MDD < PSK < Pseudopolynomial Algorithm
- The difference in accuracy between MDD and PSK will become larger as the number of jobs increase.
- Both the two heuristics become less accurate when the number of jobs is large.
- PSK will perform much better than MDD in accuracy especially when the tardiness factor is large.
- Overall, PSK has an average 32.3% error rate; MDD has an average 45.6% error rate. Neither does these two heuristics have very good performance. While considering their efficiencies in computational efforts, PSK would still be applicable in real applications.

Thank you