



# PRODUCTION SCHEDULING PROJECT

Hospital Emergency Room Scheduling

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# Project Description ... never been studied in class

- Simulated from limited data (4 Sundays) gathered at St. Luke's Hospital ER
  - ↳ ▪ Determined the distributions for each process to represent the situation.
    - ↳ ▪ Defined scheduling problem and proposed solutions
      - ↳ ▪ Conducted computational analysis and analyzed results in MATLAB

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- Processes in sequence:
    - Triage – Patients see a nurse first
    - Treatment – After receiving treatment by a doctor, patients leave the ER
      - Nurse/Doctor's processing times depend on the criticality of each patient
  - Facilities:
    - 2 treatment rooms in the ER
    - 1 nurse
    - 2 doctors
  - Patients (jobs):
    - Arrive according to Poisson distribution / Exponential arrival rate
    - 5 criticalities: (5-Critical; 4-High; 3-Medium; 2-Low; 1-Clear/non-critical)
    - Death metrics: determined by release time and a uniformly distributed random variable with expectation that's associated with criticality

# Model Assumptions

- Patients arrive according to exponential distribution with  $\lambda = 0.2$ 
  - $t_A = -\log(\text{rand})/\lambda$
- Each type of patient has a determined chance to arrive
  - 10% cleared patient; 20% low criticality  
20% medium criticality; 30% high criticality; 20% critical
- Processing times:
  - Nurse processing time is random with expected duration of 10 minutes for all patients
  - Doctor processing time depending on criticality of patients
    - Uniformly distributed within doctor processing time bracket, i.e. critical patients has processing time between 75 -95 minutes.
- Death Metrics:
  - Releasing time + Processing time with doctor + Scalar +  $10 * \text{Rand}()$ \*

\*  $\text{Rand}()$  has expected values which vary with criticality of each type of patient and range is between +/-5

# Scheduling Problem Setup –

$$J_2 | R_j | \sum U_j$$

- Modeled after a Job Shop model

- 5 Job types

Job 1 – criticality 0 (cleared) – 10%	Job 2 – criticality 1 (low) – 20%
Job 3 – criticality 3 (Medium) – 20%	Job 4 – criticality 4 (High) – 30%
Job 5 – criticality 5 (Critical) – 20%	

- 2 Machines:

- triage – Machine 1
- treatment – Machine 2

- Death Time Metric: 50 % chance                      100% chance

Job 1	Infinity	Infinity
Job 2	$R_j + A_{90} + P_{22}$	$R_j + A_{180} + P_{22}$
Job 3	$R_j + A_{36} + P_{23}$	$R_j + A_{180} + P_{22}$
Job 4	$R_j + A_{18} + P_{24}$	$R_j + A_{36} + P_{24}$
Job 5	$R_j + A_9 + P_{25}$	$R_j + A_{18} + P_{25}$

$R_j$ : Patients arrival time

$U_j$ : Number of dead patients from the system

$A_g$ : random variable constant with expected value of  $g$ , ranging from  $\pm 5$ .

# Proposed Algorithms

- First In First Out :

- Prioritize the patients with earliest arrival time
- Process earliest arrived patient when machine becomes available
- Same for both nurse queue and doctor queue (sorted on arrival time)

- Highest Criticality First / EDD:

- Assign priority to the patients with the highest criticality
- Higher criticality is associated with earliest time in death metrics, both for 50% and 100%

Job 2	$85 + 10 * \text{rand}()$	$175 + 10 * \text{rand}()$
Job 3	$31 + 10 * \text{rand}()$	$67 + 10 * \text{rand}()$
Job 4	$13 + 10 * \text{rand}()$	$31 + 10 * \text{rand}()$
Job 5	$4 + 10 * \text{rand}()$	$23 + 10 * \text{rand}()$

- Advanced Expected Death EDD:

- Comparing completion time with death metrics for patients in queue
- Discard the patients with 100% certainty to die regardless received treatment or not, then advance the process by taking the next patient in queue
- Feasible Lower Bound

# FIFO Model & Output Analysis

- Store patients in queues for doctor and nurse
- Sort doctor queue based on the arrival time
  - `[Y,I] = sort(queueD(1,:), 'descend');`  
% queueD stores patients waiting to be treated by doctor
  - Check if the patient is dead before service starts

```
if (serviceD1(3, Ncustomers+1) > queueD(4, 1))      %If job is dead before service
    serviceD1(2, Ncustomers+1) = 0.001;           %set dead jobs as cleared
    serviceD1(3, Ncustomers+1) = serviceD1(1, Ncustomers+1) + serviceD1(2, Ncustomers+1);
                                                    %end time
    NDeath = NDeath+1;
end
```

- Take the next on sorted queue when additional machine becomes available

Result: ( 20 exits in each simulation; 1000 simulations)

Number of 100% Death	Number of 50% Death	Total Number Completed	Make Span (minutes)
7.08	2.12	20	299.55

# Highest Criticality First/EDD

Similar set up as the FIFO except when queueD > 0 & doctor or nurse becomes available, use a different sorting schema

```
if (NextDoctorAvailable == 1)           % if doctor 1 is the next available doctor
...
    EDDQueueD = EDDQueueD - 1;
    [Y,I] = sort(queueD(1,:), 'ascend'); % sort by criticality
    queueD = queueD(:,I);
    [Y,I] = sort(queueD(2,:), 'descend'); % sort by arrival time
    queueD = queueD(:,I);
```

Result: ( 20 exits in each simulation; 1000 simulations)

Number of 100% Death	Number of 50% Death	Total Number Completed	Make Span (minutes)
7.28	2.08	20	299.83

# Advanced EDD Output Analysis

```
% check is job dead before arrival
if serviceD1(1,Qcustomers+1)>queueD(4,1)      %If job is dead before service
    serviceD1(2,QCustomers+1) = 0.001;        %set dead jobs as cleared
    serviceD1(3,QCustomers+1) = serviceD1(1,QCustomers+1) + serviceD1(2,QCustomers+1); %end
    time
    NDeath = NDeath+1;
end

% check is the job expected to die before service end
if serviceD1(3,QCustomers+1)>queueD(4,1)
    serviceD1(2,QCustomers+1) = 0.001;        %set dead jobs as cleared
    serviceD1(3,QCustomers+1) = serviceD1(1,QCustomers+1) + serviceD1(2,QCustomers+1);
    %end time

    NDeath = NDeath+1;
end
```

Result: ( 20 exits in each simulation; 1000 simulations)

Number of 100% Death	Number of 50% Death	Total Number Completed	Make Span (minutes)
4.19	3.07	20	272.32



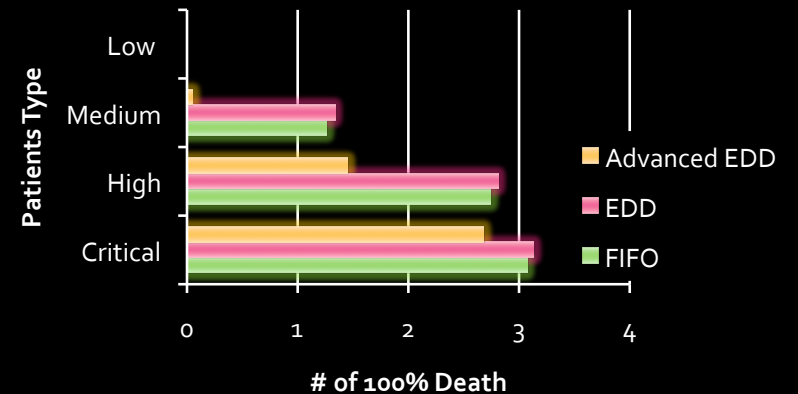
# Results & Conclusions

	Number of 100% Death	Number of 50% Death	Make Span (minutes)
FIFO	7.08	2.12	299.55
EDD	7.28	2.08	299.83
Advanced EDD	4.19	3.07	272.32

- Advanced Expected Death EDD output can be used as a feasible LB
- EDD is not necessarily better than FIFO
- Make Span from FIFO and EDD are very close to each other
- EDD can result in large queues
  - i.e. for a simulation with 20 jobs, EDD can end up having >100 people in queue

- Advanced EDD has the least number of death in every type
- In all three algorithms, most death happens in "Critical"
- EDD yields higher number of death in all three types of patients

Distribution of Death in Each Type



# Also keep in mind ... sanity check

- Advanced EDD = Let patients die before being seen by doctors or nurses?
  - How practical that really is?
- Who determine the “criticality” of the patients?
  - Possibilities of making mistake on assigning “criticality” to patients?
  - How do we know before patients going through arbitrage?
- Advanced EDD discard 100% certainty death, what about 50% death?
- Offline Scheduling
  - Know the arrival of each patients then schedule the jobs
  - Will this be a better LB?



# Thank you!